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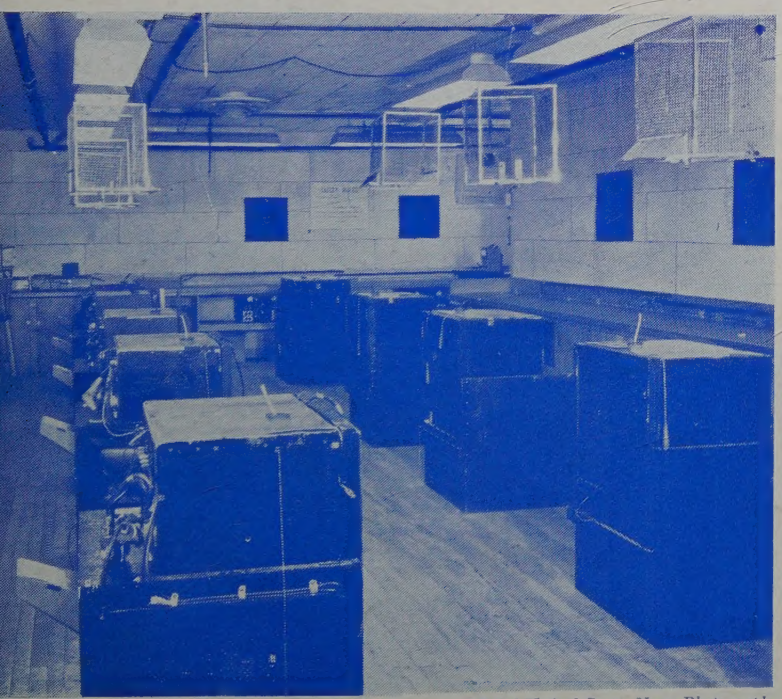

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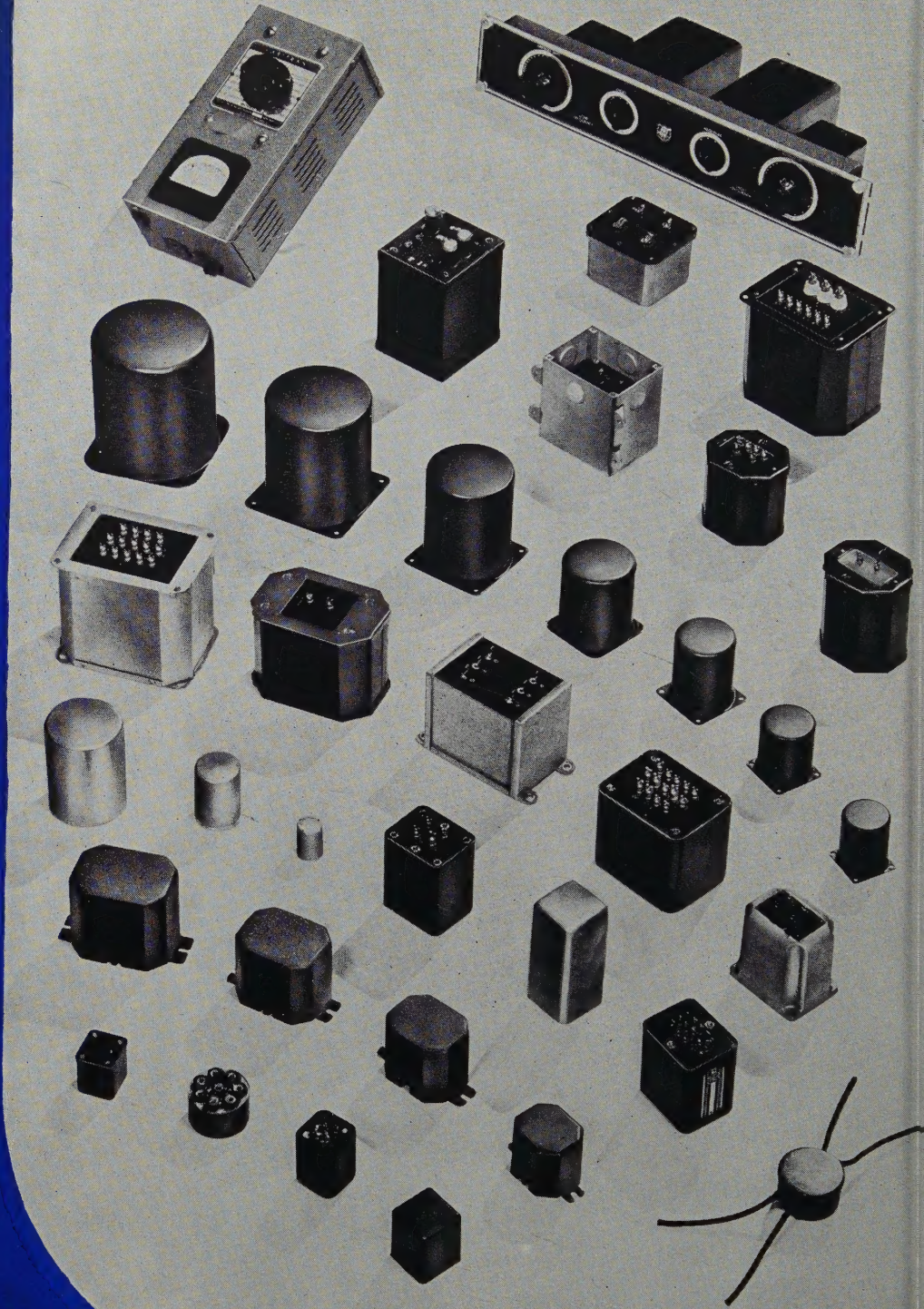

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The readers of the PROCEEDINGS OF THE I.R.E. are interested in the viewpoints of commercial leaders in the radio-and-electronic industry, in relation to technical and production trends under wartime and post-war conditions, as well as the part which the engineer should and can play in the corresponding activities. Accordingly the PROCEEDINGS here presents a "guest editorial," as received from its author, the president of the Sperry Gyroscope Company.

The Editor

Electronic Applications

R. E. Gillmor

For over twenty-five years the Sperry Gyroscope Company has specialized in three main fields of endeavor. First, marine-navigation instruments such as gyro compasses; second, aviation instruments such as blind-flying instruments and automatic gyro pilots; third, military equipment such as antiaircraft searchlights, sound locators, and bombsights.

There is nothing in the above list of products and instruments which would suggest any direct interest in electronics, and yet it is just because of that obscure connection that I am pleased to accept the opportunity to write this brief editorial for the PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS. My story is one which will show the naturalness and effectiveness of electronics in moving into these widely separated fields as a tool which has improved every one of the products.

It was twelve years ago that, in our search for a better method than trolley and contact for picking off the position of the compass gyroscope without pushing, dragging, or even touching it, we first experimented with electronics to do the job. The solution was a new type of amplifier which might be called a torque amplifier. The signal, made by the mere change of position of a small bar of iron carried on the gyro, caused almost zero torque and hence no disturbance to the gyro, but the output of the amplifier drove an azimuth motor with sufficient torque to operate repeater compass transmitters, metal mike, etc., with all the accuracy of the master gyro.

So electronics first stepped into the Sperry Company as a new tool which solved one of our tough gyro problems.

From there electronic power control moved onto our searchlights to train them and elevate them from remote selsyn motors. In aviation, although radio came aboard the airplane very early for communication purposes, electronics for control purposes had to wait the day when other more orthodox methods had reached their limitations. Sperry automatic pilots went through various stages of pneumatic bellows, mechanical clutches, hydraulic valves, and electric contactors. These well-known methods did the job of automatic piloting satisfactorily until recently when airplanes have become so large and, at the same time military needs called for such precision, that the only answer again has been to go to electronics for the huge and precise step of amplifying small gyro signal to large control force.

These examples show how, step by step, electronics has taken over certain functions in our products which have improved their performance but have not changed their names.

It is, however, even reaching farther than this. Some ten years ago, we decided after many experiments that the 60-inch high-intensity searchlight was as powerful as was practical to make. Larger lights gave almost no increase in range. To surpass the searchlight, we must have something better than light. Sound locators, we also realized were very limited in their range and could never be precision instruments; we needed something better than sound. After considerable search, we had found that the best solution probably lay in the field of radio. I need not go into further details. This again was a case of electronics coming in to aid or even supersede our antiaircraft weapons and accomplish the same functions better than by the old methods.

We never entered the electronic business as a distinct field. Electronics, on the other hand, entered our business, and our products have been vastly improved by its applications. We see no end in sight of utilizing more and more applications of this new tool, and our research laboratory is now as well equipped for electronics research as it has been for gyroscopics, hydraulics, and many other older fields.

Harold Alden Wheeler

Harold Alden Wheeler was born in St. Paul, Minnesota, in 1903. He developed an early interest in radio engineering and in 1925 he was graduated with the B.S. degree in physics from George Washington University. This was followed by postgraduate work at Johns Hopkins University where he specialized in the electrical field. His research work at this university won him election to Sigma Xi and Gamma Alpha.

In 1922 he constructed a neutralized tuned amplifier and in 1925 assembled an eight-tube superheterodyne receiver in which the novel features of diode linear detection and automatic volume control were included. While studying at Johns Hopkins University, Wheeler's research projects included a theoretical study of wave filters, an experimental test of the thermal noise formula, the detection of weak modulated light beams by a photoelectric cell and selective detector, and the design of a pulse amplifier for operation on the heart-action currents, problems closely related to those encountered in present-day radio and television work.

In 1922, Mr. Wheeler accidentally met Professor Alan Hazeltine with whom he found a common interest in neutralized amplification. He worked summers with Professor Hazeltine and continued with the Hazeltine Corporation from its inception in 1924. He was put in charge of the Bayside Laboratory in 1930 and was later made Vice President of what is now the Hazeltine Electronics Corporation, with laboratories located at Little Neck, Long Island.

His scientific contributions have been numerous and varied. He developed special testing equipment, such as the piston alternator; he worked out a simple inductance formula for solenoid coils; his theoretical studies of distortion and wide-band amplifiers were the basis for the award to him in 1940 of the Morris Liebmann Memorial Prize; in his studies of frequency modulation, he substituted for the accepted attack of sine-wave modulation with its multiple sidebands, a generalized modulation, which yields simple relations of greater utility; he unified the formulas for the skin effect; and recently he has been developing charts and formulas for streamlining many phases of radio design. Much of his work has appeared in published papers in the PROCEEDINGS.

Mr. Wheeler has been active in Institute affairs from the time he attended Johns Hopkins. His work during the summers of 1921 and 1922 in the radio laboratory at the Bureau of Standards gave him his foundation for subsequent standardization work in the Institute. He served as chairman of the Technical Committee on Radio Receivers for the 1933 and 1938 reports, and he is now chairman of the Standards Committee. He was a member of the Papers Committee, and now is a member of the Board of Editors and a director of the Institute. He joined the Institute of Radio Engineers as an Associate in 1927, transferred to Member grade in 1928, and to Fellow in 1935.

Radio-Frequency Heating Applied to Wood Gluing*

R. A. BIERWIRTH†, NONMEMBER, I.R.E., AND CYRIL N. HOYLER†, ASSOCIATE, I.R.E.

Summary—**Dielectric heating by radio-frequency power has been established[1,2] as a practical means for rapidly gluing wood with thermosetting adhesives. To the radio engineer, such an application involves the correlation of power concentration, frequency, voltage gradient, dielectric constant, and power factor for the determination of reasonable operating parameters. An equation is developed showing the interdependence of these factors. Pertinent data on the dielectric properties of several kinds of wood are included covering a suitable range of frequency and for the temperatures and moisture contents commonly encountered.**

The nonuniform heating likely to occur in large presses because of standing waves on the electrodes can be eliminated by "multiple tuning" in which each of several inductors tunes a section of the press electrodes to parallel resonance.[3] A coupling network is evolved for feeding a tuned press so that variations in press capacitance during the gluing cycle become unimportant to the loading of the oscillator. Means for measuring the temperature at the glue line during the application of radio-frequency power are discussed.

I. Introduction

THE GENERATION of heat in dielectric materials by strong alternating electric fields has long been evident to radio engineers dealing with high power at the higher frequencies. However, when a poor dielectric material is purposely subjected to such a field, heat will be generated rapidly and uniformly throughout, in a manner that can be closely controlled. One industrial application of such radio-frequency heating is made in the wood-gluing industry where the recent increase in the use of laminated wood, as a material of construction, has accentuated the need for rapidity in curing the adhesives commonly used for bonding. Although gluing as an art and its application to the manufacture of plywood may be traced to ancient Egypt, it was not until the introduction of hot-platen presses about the time of World War I that the way was first pointed to a shorter gluing cycle and greater durability of the finished product. During the last decade the development of thermosetting adhesives of the synthetic-resin group has broadened the demand for a means of heating the glue line quickly and uniformly so that the inherent advantages of these modern adhesives could be most fully realized. Hot-plate presses can be used effectively where the glue lines are close to the surface, but when heat must be conducted through thick sections, the heating cycle becomes too long and excessive drying or scorching of the wood arises as a problem. Dielectric heating by radio frequency simplifies this problem since heat is generated uniformly throughout the material and thus glue lines some distance from the surface may be as readily heated as those nearer the surface. In a paper by G. H. Brown,[4] a theoretical examination has been made of the heat-conduction problems encountered in hot-press wood gluing where the heating energy is supplied either by hot plates or by radio-frequency power. Basic relationships between heating time, temperature rise and power concentration are also established in his paper. For the proper application of radio-frequency heating to presses designed for wood gluing some information must be assembled on the electrical properties of wood at several frequencies as well as the means by which the radio-frequency energy may be coupled to electrodes in the press. It is the object of this paper to discuss some of the practical aspects of radio-frequency heating as applied to wood gluing.

II. Thermosetting Glues

The thermosetting glues most widely used in the woodworking industry today are either of the synthetic urea-resin type or of the synthetic phenolic-resin type.

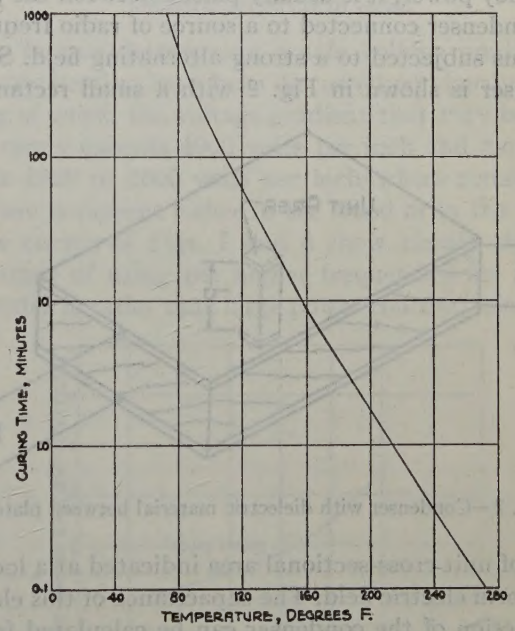

Fig. 1—Relationship of curing time and temperature for a catalyzed urea-resin glue.

Urea-resin glue can be obtained in the form of a liquid ready for use or in the form of a powder which must be dissolved in water before using. Fig. 1 shows a typical curing-time—temperature curve for a catalyzed glue of this type. It will be noted that for a temperature of 220 degrees Fahrenheit, the curing time is about one minute while at a room temperature of 75 degrees

* Decimal classification: R590. Original manuscript received by the Institute, March 9, 1943.

† RCA Laboratories, Princeton, New Jersey.

[1] "High-frequency heat used to make plywood," *Elec. World*, vol. 117, p. 2240; June 27, 1942.

[2] J. P. Taylor, "Heating wood with radio frequency power." Presented, annual fall meeting, American Society of Mechanical Engineers, Woodworking Section, Rochester, New York, October 12, 1942.

[3] R. A. Bierwirth, United States Patent 2,308,043, 1943.

[4] George H. Brown, "Heat-conduction problems in presses used for the gluing of wood," PROC. I.R.E., this issue, pp. 537–548.

Fahrenheit the curing time is 13 hours. Although the curing time can be further reduced by the addition of more catalyst, the glue may then set too quickly at room temperature to be of practical use. Pressures of 150 pounds per square inch or greater should be maintained during the gluing cycle so that irregularities in the adjacent wood surfaces may be eliminated. If the bond is properly cured it is highly water resistant at ordinary temperatures but not at elevated temperatures. Urea-resin glue is lower in price than phenolic glue and can be extended by mixing with wheat or rye flour.

Phenolic-resin glue can be obtained in the form of a liquid, a film, or a powder. The powder must be dissolved in water or alcohol before use. For best results this type of glue generally requires a bonding temperature of 280 to 300 degrees Fahrenheit, maintained for several minutes with an applied pressure as great as the wood will stand without damage. The resulting bond is stronger than the wood and resistant to water even at its boiling point. Specific information on the curing time-temperature relation for any particular glue can be obtained from the glue manufacturer.

III. Dielectric Heating

Where a dielectric material is to be heated by radio-frequency power, it is usually placed between the plates of a condenser connected to a source of radio frequency and thus subjected to a strong alternating field. Such a condenser is shown in Fig. 2 with a small rectangular

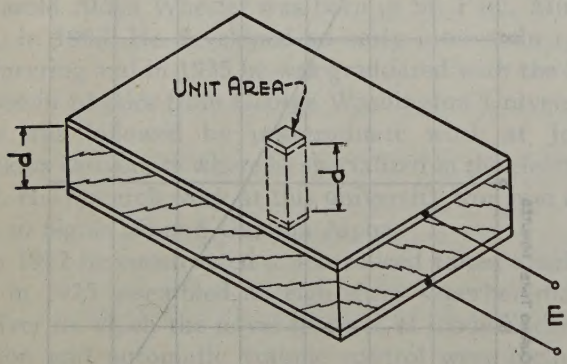

Fig. 2—Condenser with dielectric material between plates.

prism of unit cross-sectional area indicated at a location of uniform electric field. The capacitance of this elementary section of the condenser can be calculated from

$$C = 0.225\epsilon/d \text{ micromicrofarads} \qquad (1)$$

if the dielectric constant ϵ of the material between the plates is known and the dimensions are given in inches.

When a radio-frequency voltage E is applied across the plates of this condenser the power absorbed by the rectangular prism is

$$P = (E^2/(\sqrt{R^2 + X^2}))\ (\cos\theta) \text{ watts} \qquad (2)$$

where R and X in ohms are, respectively, the resistive and reactive components of the impedance for this elementary condenser and $\cos\theta$ is the power factor.

Rewriting (2) in the following manner

$$P = (E^2/X)\cdot(X/(\sqrt{R_2 + X^2}))\ (\cos\theta) \text{ watts} \qquad (3)$$

shows the way to the further modification

$$P = (E^2/X)(\sqrt{1 - \cos^2\theta})\ (\cos\theta) \text{ watts} \qquad (4)$$

since

$$X/(R^2 + X^2) = \sin\theta = \sqrt{1 - \cos^2\theta}.$$

By using the computed value of C from (1), the value of X in this equation for a frequency of f megacycles may be found from $X = 1/2\pi fC$ and inserted into (4) which then can be written

$$P = (1.415 \times 10^{-6}\ f\epsilon\ E^2)/d\sqrt{1 - \cos^2\theta}\cos\theta \text{ watts}. \qquad (5)$$

For the power concentration P_c, in watts per cubic inch, (5) reduces to

$$P_c = 1.415 \times 10^{-6}\ f\epsilon\ (Eg)^2\sqrt{1 - \cos^2\theta}\cos\theta$$
$$\text{watts per cubic inch} \qquad (6)$$

in which E_g is the voltage gradient across the condenser in volts per inch.

In a specific application of dielectric heating, the permissible voltage across the condenser is limited by such factors as configuration of electrodes, dielectric strength, presence of moisture with possible generation of steam, or irregularities in the wood such as knots. Since the power concentration will usually be fixed by oscillator output and volume of material being heated, a helpful revision of (6) is its solution in terms of the voltage gradient E_g. Thus:

$$E_g = 841[P_c/(f\epsilon\sqrt{1 - \cos^2\theta}\cos\theta)]^{1/2} \text{ volts per inch}. \qquad (7)$$

It has been found, with reference to wood, that these last two equations may be somewhat simplified and the next section shows how such simplification may be effectively used.

IV. Wood as a Dielectric

It is apparent from (6), for example, that at a given frequency the power absorbed by a dielectric is directly related to its dielectric constant and the power factor. For wood these values are not constant but change with frequency as shown in the curves of Figs. 3 and 4. To assemble the data represented by these curves, measurements were made on a number of samples of fir plywood, Sitka spruce, and walnut all of which were 4 inches long, $2\frac{1}{4}$ inches wide, and $\frac{1}{4}$ inch thick. A type 170-A Q meter with a condenser of the form shown in Fig. 5 was used for these measurements. Attention must be drawn to the fact that the curves of Figs. 3 and 4 represent an average of a number of measured values as considerable variation is evident between samples. Such evidence is found in the behavior of several samples, each with $6\frac{1}{2}$ per cent moisture content, that were taken at random from a stock of spruce. Their measured power factor varied from 7 to $8\frac{1}{2}$ per cent and the dielectric constant varied between 2.6 and 3.2. Typical curves are shown in Fig. 6 for the relationship of moisture content with dielectric constant and power factor at 45 megacycles for spruce.

In view of the wide variations in the electrical

properties of these woods it may appear hopeless to find any equitable basis by which (6) or (7) may be adapted to simplified use. Nevertheless, for a first approximation, these curves show that for a moisture content of from 4 to 7 per cent the product of ϵ and $\cos\theta$ is found to be reasonably close to 0.177 by which (6) may be simplified to

$$P_c = 25 \times 10^{-8} (E_g)^2 f \text{ watts per cubic inch} \qquad (8)$$

and then (7) becomes

$$E_g = 2000\sqrt{P_c/f} \text{ volts per inch.} \qquad (9)$$

The restriction as to moisture content may be justified

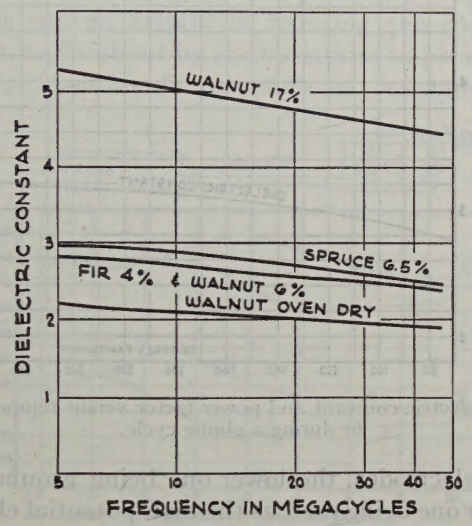

Fig. 3—Effect of frequency on dielectric constant of several species of wood at indicated moisture content.

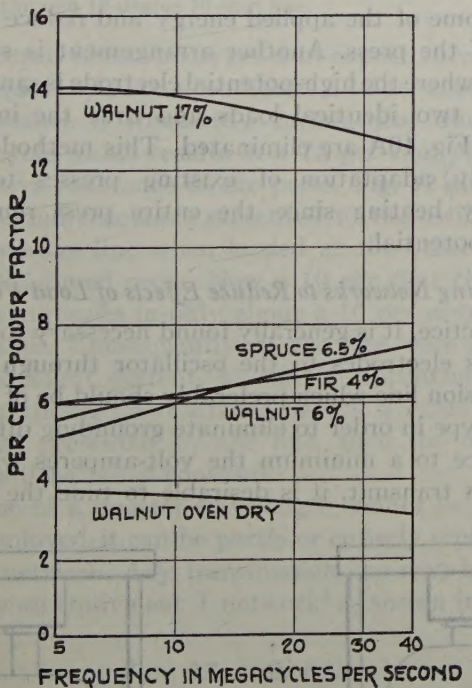

Fig. 4—Effect of frequency on power factor of several species of wood at indicated moisture content.

on the grounds that this range is representative of values which are frequently encountered in the wood-gluing industry. Allowing the approximations that have been made it can then be said that the voltage gradient across a condenser with wood between its plates is directly proportional to the square root of power concentration and inversely proportional to the square root of frequency. Curves showing the relationships expressed by (8) and (9) are given in Figs. 7 and 8, respectively. Although these curves relate specifically to Sitka spruce they may be used for similar woods having moisture contents normally encountered. In actual gluing operations the power concentration has varied from 5 to 75 watts per cubic inch and the voltage gradient from 1000 to 4000 volts per inch at various frequencies ranging from 1.5 to 45 megacycles.

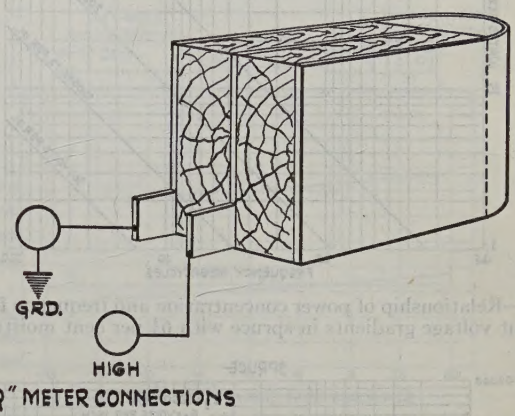

Fig. 5—Condenser for use with Q meter in determining dielectric properties of wood.

The factors determining a safe voltage gradient for dielectric heating have been discussed previously. In the gluing of wood, the voltage gradient that may be safely used rarely exceeds 4000 volts per inch and may be as low as 1000 to 2000 volts per inch where considerable moisture is present either in the wood or in the glue.

The curves of Figs. 7 and 8 show clearly the great advantage of using the higher frequencies for heating dielectrics in order that large power concentrations may

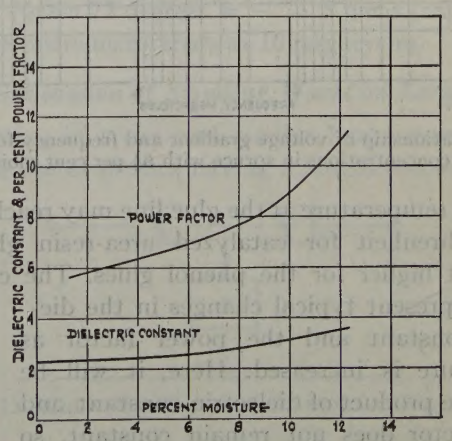

Fig. 6—Variation of power factor and dielectric constant with moisture content for spruce at 45 megacycles.

be used without excessive voltage gradients. Unfortunately, high power at high frequency becomes difficult to generate so that the actual choice of operating frequency may be a compromise. Practical experience has shown that frequencies in the 5- to 15-megacycle range

can be economically generated so that their use for wood-gluing applications can be readily justified.

Another factor that affects these electrical properties of the wood is temperature. In the course of a gluing

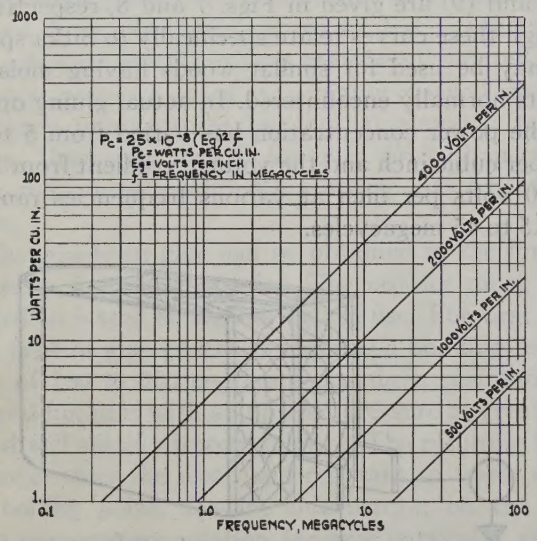

Fig. 7—Relationship of power concentration and frequency for different voltage gradients in spruce with 6½ per cent moisture.

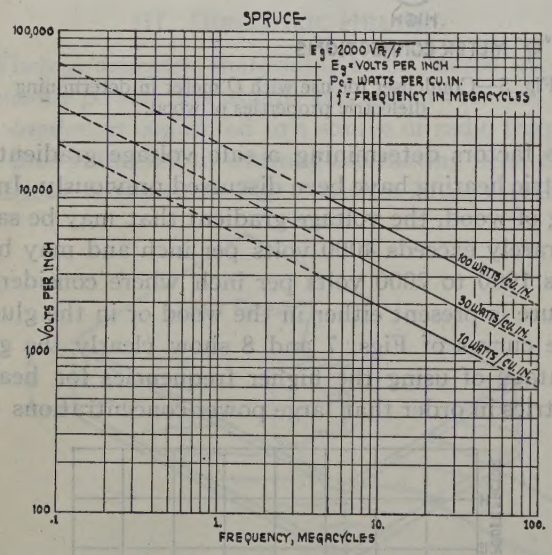

Fig. 8—Relationship of voltage gradient and frequency for different power concentrations in spruce with 6½ per cent moisture.

cycle, the temperature at the glue line may reach 240 degrees Fahrenheit for catalyzed urea-resin glues and somewhat higher for the phenol glues. The curves of Fig. 9 represent typical changes in the dielectric constant and the power factor as temperature is increased. Here, it will be noted, the product of dielectric constant and power factor does not remain constant, so that a change in the voltage gradient and power concentration will become apparent. The capacitance will also change with dielectric constant, so that the consequent detuning may have an undesirable influence on the oscillator. Means for reducing the effect of this condition will be discussed later.

V. Coupling Problems

A. Electrode Arrangements

Two typical electrode arrangements, by which radio-frequency power may be applied to wood-gluing presses, are shown in Fig. 10. In Fig. 10A the press plates are

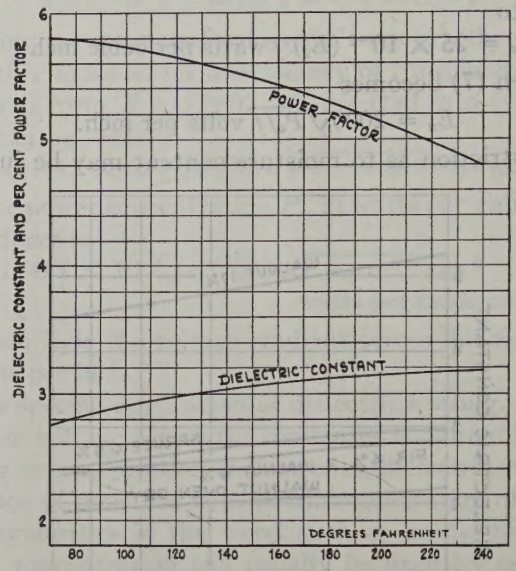

Fig. 9—Dielectric constant and power factor versus temperature for fir during a gluing cycle.

used as electrodes, the lower one being grounded and the upper one being used as the high-potential electrode, which must therefore be insulated from the rest of the press. Unless this insulation is of low-loss material it will absorb some of the applied energy and reduce the efficiency of the press. Another arrangement is shown in Fig. 10B where the high-potential electrode is sandwiched between two identical loads and thus the insulation losses of Fig. 10A are eliminated. This method permits the ready adaptation of existing presses to radio-frequency heating since the entire press remains at ground potential.

B. Coupling Networks to Reduce Effects of Load Variations

In practice, it is generally found necessary to connect the press electrodes to the oscillator through a short transmission line which preferably should be of the concentric type in order to eliminate grounding difficulties. To reduce to a minimum the volt-amperes which the line must transmit, it is desirable to tune the press to

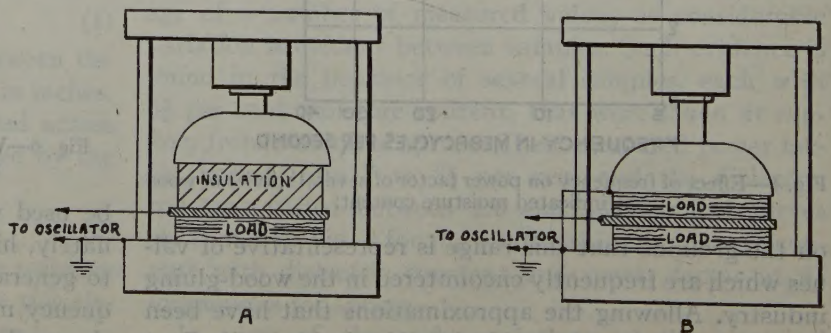

Fig. 10—Methods of applying radio-frequency power to wood-gluing presses.

parallel resonance by connecting an inductance of the proper value in shunt with the electrodes. However, Fig. 9 shows that the dielectric constant of wood varies with temperature so that the capacitance of the press will change during the gluing cycle thereby detuning the circuit and changing the oscillator loading. If the oscillator is self-excited, the frequency will shift to make partial compensation for the detuning, but unless the oscillator tank circuit is very closely coupled to the press the change in loading may be appreciable.

The use of a quarter-wave transmission line, coupled to the oscillator through a comparatively large reactance, has proved helpful in reducing the changes in loading brought about by the variations in press capacitance. Fig. 11 shows the reactance and resistance of the

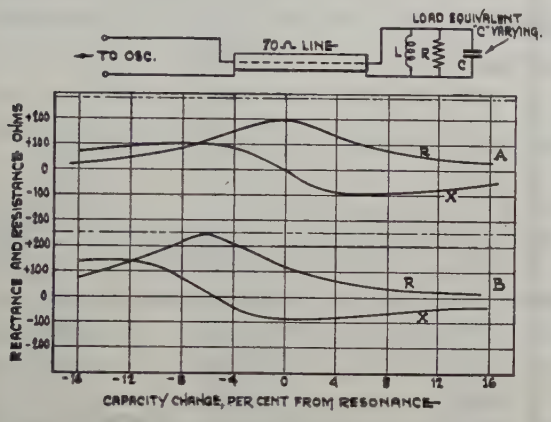

Fig. 11—Effect of capacitance change in the load on the resistance and reactance presented to the oscillator when coupled: *A*, directly, and *B*, through 18-degree 70-ohm line.

parallel-tuned press versus per cent capacitance change. Under typical operating conditions a 10 per cent change in capacitance is frequently encountered during the heating cycle which results in a 75 per cent change in the effective resistance of the press. Fig. 12 shows the resistance and reactance presented to the oscillator by a quarter-wave line when loaded at the other end by the parallel-tuned press. Now a 10 per cent change in capacitance results in only about a 10 per cent change in resistance. Consequently, if the line were supplied with a constant current, as can be approximated by feeding it through a reactance having a value 5 to 10 times that presented by the line, then the oscillator loading will be fairly constant.

If a line of a quarter wavelength cannot be conveniently employed, it can be partly or entirely replaced by a T or π network. Any transmission line may be represented by an equivalent T network[5] as shown in Fig. 13 in which

$$Z_1 = 2Z_c \tanh P/2 \qquad (10)$$

$$Z_2 = Z_c/\sinh P \qquad (11)$$

where Z_c = characteristic impedance of the line

P = propagation factor of the line in the form $\alpha + j\beta$

α = attenuation factor negligible for a short line

β = phase shift in degrees

Thus for a short line (10) becomes

$$Z_1 = 2Z_c \tanh j\beta/2$$
$$= 2Z_c \sinh j\beta/(\cosh j + 1)$$
$$= j2Z_c \sin \beta/(\cos \beta + 1) \qquad (12)$$

and (11) becomes

$$Z_2 = Z_c/\sinh j\beta.$$
$$= -jZ_c/\sin \beta. \qquad (13)$$

In one laboratory application, the concentric line feeding the press was 18 degrees long for an operating fre-

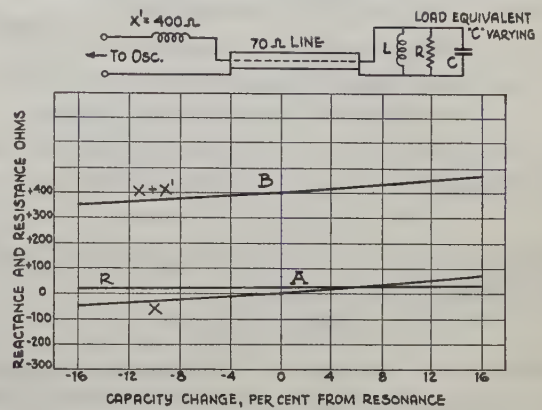

Fig. 12—Effect of capacitance change in the load on the resistance and reactance presented to the oscillator when coupled through a 90-degree line. *A*, resistance and reactance of load and *B*, reactance of load fed through same 90-degree line with 400-ohm series reactor (resistance remains the same as in *A*).

quency of 10 megacycles and had a characteristic impedance of 70 ohms. To "lengthen" this line to 90 degrees required the addition of a network which had a phase shift of 90 − 18 = 72 degrees. From (12) and (13)

$Z_1/2$ = 70 sin 72 degrees/(cos 72 degrees + 1)
= 50.8 ohms (inductive)
= 0.8 microhenry at 10 megacycles

Z_2 = − 70/sin 72 degrees = − 73.5 ohms (capacitive)
= 218 micromicrofarads at 10 megacycles.

C. The Elimination of Standing Waves on Large Presses

The gluing of spars and similar long structures requires the use of presses having a length of twenty feet

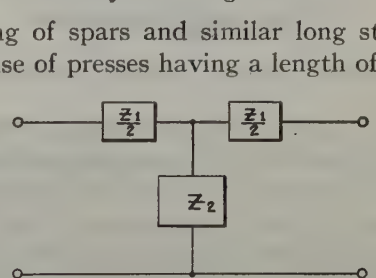

Fig. 13—Equivalent T-network for a transmission line.

or more. At mentioned above, to obtain a reasonably short gluing cycle and realize freedom from flashovers it is desirable to use a frequency in the order of 10 megacycles per second. However, when power at such a frequency is applied to long electrodes, the voltage distribution along the electrodes will be nonuniform because of standing waves unless proper precautions are taken.

[5] E. A. Guillemin, "Communication Networks," John Wiley and Sons, Inc., New York, N. Y., 1936, vol. II, p. 178.

The long electrodes may be considered as an unterminated transmission line and the standing waves thereon will have a length dependent on the frequency and the dielectric constant of the material between the electrodes.

In free space, the wavelength is given by the expression

$$\lambda_1 = v_1/f \text{ meters} \tag{14}$$

where v_1 is the wave velocity in free space (3×10^8 meters per second) and f is the frequency in cycles per second. The ratio of the wave velocity v in any other medium to that in free space is

$$v/v_1 = \sqrt{(\mu_1\epsilon_1)/\mu\epsilon} \tag{15}$$

where $\frac{\mu_1}{\mu} = \frac{\text{the permeability of free space}}{\text{the permeability of the medium}} = 1$ for wood

ϵ_1 = the dielectric constant of free space = 1

ϵ = the dielectric constant of the medium.

The wavelength, when the press is loaded with wood, can be obtained by combining (14) and (15) as

$$\lambda = (3 \times 10^8)/(f\sqrt{\epsilon}) \text{ meters.} \tag{16}$$

For the wavelength in feet and the frequency in megacycles this expression becomes

$$\lambda = 984/(f\sqrt{\epsilon}) \text{ feet.} \tag{17}$$

If voltage at a frequency of 14 megacycles is applied to the plates at one end of a press 20 feet long as shown

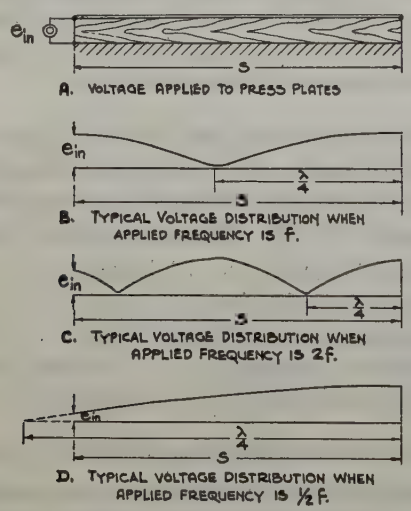

Fig. 14—Voltage distribution on long presses for several frequencies.

in Fig. 14A, then the standing wave of voltage will be as shown in Fig. 14B. The standing waves will have maxima at the open end and at a half wavelength and multiples thereof from the open end. The voltage will approach zero at points a quarter wavelength from the maxima. Figs. 14C and 14D show the standing waves for double and half the frequency, respectively, of Fig. 14B. In order to get reasonably uniform voltage on the plates when they are fed in this manner, it is apparent that the frequency applied to the press must be such that a quarter wavelength is considerably greater than the length of the press. For this condition the ratio of the minimum to the maximum voltage appearing on the press plates is given by

$$E_{min}/E_{max} = \cos(360s/\lambda) \tag{18}$$

where s is the length of the press in feet. Eliminating λ from (17) and (18) and solving for the frequency f yields

$$f = 2.74/(s\sqrt{\epsilon}) \cos^{-1}(E_{min}/E_{max}) \text{ megacycles} \tag{19}$$

where $\cos^{-1}$ is expressed in degrees. Since it is desirable to keep the ratio of the minimum to the maximum voltage 0.9 or greater, long presses would require such a low frequency for a safe voltage gradient that the power concentration as obtained from Figs. 7 or 8 would be insufficient for a reasonably short gluing cycle.

A relationship between the length of a press and percentage variation in voltage or power due to standing waves is given by the graph of Fig. 15 for a number of

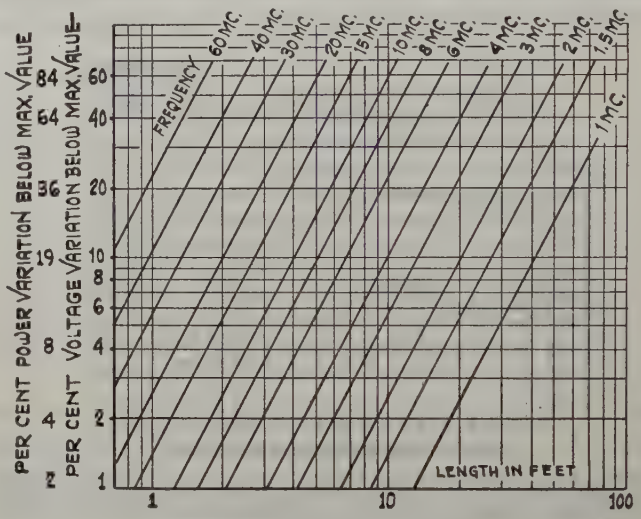

Fig. 15—Variations in power and voltage due to standing waves on long electrodes fed at one end.

different frequencies. If the voltage is applied to the plates at the center, each half of the press may be considered separately and the frequency for a given voltage variation is twice that permissible for the case where the voltage is applied at one end. This suggests a means whereby a more uniform voltage distribution may be realized. By feeding the press simultaneously at the centers of two or more equal divisions of the plates, a more uniform voltage distribution results, as shown in Fig. 16. As the number of divisions is increased, the frequency that may be used for a given voltage variation increases proportionately. The application of identical voltages at various points along the press plates by this method presents a somewhat difficult problem involving the use of a properly proportioned network of transmission lines. However, the improved voltage distribution resulting from this use of multiple feed lines can be more readily obtained by multiple tuning; that is, by replacing each feed line with an inductor of the proper value to tune its division to parallel resonance, as illustrated in Fig. 17. The transmission line from the oscillator may be connected to the plates at the end or any other convenient point. The spacing of these tuning elements d for any desired voltage ratio and frequency may be obtained from (19), noting that the value of s

in that expression is equal to one half of the spacing. Thus, in terms of d, (19) becomes

$$d = 5.48/(f\sqrt{\epsilon}) \cos^{-1} (E_{min}/E_{max}) \text{ feet.} \quad (20)$$

The quantity ϵ in the above expression is the dielectric constant of the material between and around the plates.

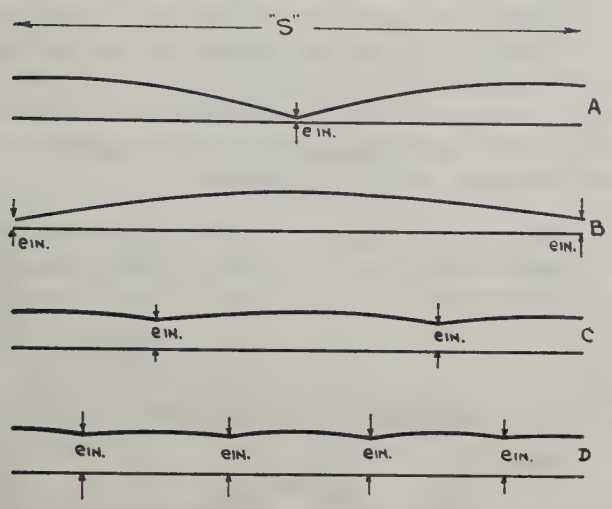

Fig. 16—Voltage distribution on press electrodes of length S for various points of feed.

If the dielectric constant of the wood between the plates is used, the value of spacing obtained will be somewhat conservative.

Since the tuning inductors placed in parallel across the electrodes must collectively tune the capacitance of the press to parallel resonance the inductance of each tuning inductor is given by

$$L = (n \times 10^6)/(4\pi^2 f^2 C) \text{ microhenries} \quad (21)$$

where n = the number of equally spaced tuning elements
f = the frequency in megacycles per second
C = the capacitance of the press plates in micro-microfarads.

At resonance the press will then present to the feed line a resistive load having a value of approximately

$$Z_r = 10^6/(2\pi f C \cos\theta) \text{ ohms.} \quad (22)$$

Two illustrative examples of the use of multiple tuning in the elimination of standing waves on press plates

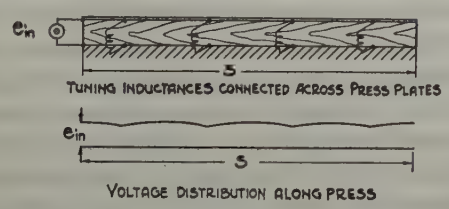

Fig. 17—Voltage distribution on press electrodes of length S fed at one end and "multiple-tuned" by inductors.

will be given. A conventional hot-plate gluing press with plates 13 feet long was adapted for the radio-frequency gluing of spruce spars. The spars were made up of three laminations each 13 feet long, 6 inches wide, and 5/16 inch thick. As shown in Fig. 18, a thick piece of fir planking was used to insulate the upper electrode from the upper pressure member of the press, and the bed of the press was used as the lower grounded electrode. An oscillator with an operating frequency of 45 megacycles was available. Without multiple tuning the standing waves which resulted from the application of power to the press at this frequency are shown by curve A in Fig. 19. Measurement of this voltage distribution was

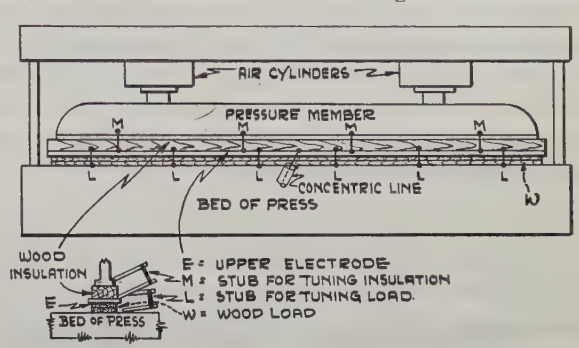

Fig. 18—Wood-gluing press adapted for radio-frequency heating at 45 megacycles.

made by taking vacuum-tube-voltmeter readings at 6-inch intervals along the press when low power was applied. The spacing of the tuning inductors used to correct this condition was calculated from (20) using the following constants:

frequency, $f = 45$ megacycles
dielectric constant for spruce, $\epsilon = 2.4$ (from Fig. 4)
power factor for spruce, $\cos\theta = 0.076$ (from Fig. 3)
length of plates, $s = 13$ feet
capacitance of plates, $C = 700$ micromicrofarads (measured)
voltage ratio, $E_{min}/E_{max} = 0.9$ (chosen value).
The spacing $d = 5.48/(45\sqrt{2.4}) \cos^{-1} 0.9 = 2.04$ feet.

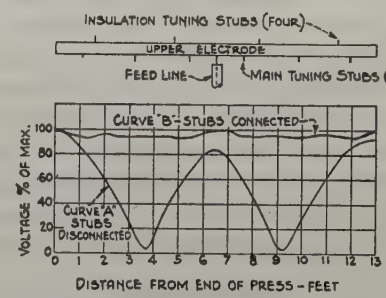

Fig. 19—Measured voltage distribution along press electrodes of Fig. 18 at 45 megacycles.

Actually six tuning elements were used with a spacing of 13/6 or 2.17 feet. The inductance value was obtained from (21) as

$$L = 6/(4\pi^2 \times 45^2 \times 700) = 0.107 \text{ microhenries.}$$

Each inductor was made by bolting a 6-inch piece of $\frac{1}{2}$-inch copper tubing to the upper and lower electrodes of the press and joining the ends by a movable strap to facilitate final tuning. With low power applied to the press, adjustment of the strap was made for maximum deflection in a thermogalvanometer loosely coupled to the inductor by a coil across its terminals. When the press is tuned to resonance its impedance is given by (22) as

$Z_r = 10^6/(6.28 \times 45 \times 700 \times 0.076) = 66$ ohms (approximate).

Curve B in Fig. 19 shows the voltage distribution with the tuning elements connected at the locations shown in the figure. It will be noted that tuning inductors were also connected from the upper electrode to the upper pressure member of the press to tune the insulation. With a power input of 28 kilowatts to the press, the temperature of the glue lines was raised to 260 degrees Fahrenheit in 2 minutes. The theoretical power requirements for no losses was calculated[4] to be 18 kilowatts, indicating an efficiency of approximately 65 per cent. This low efficiency may be largely attributed to the losses in the insulating block.

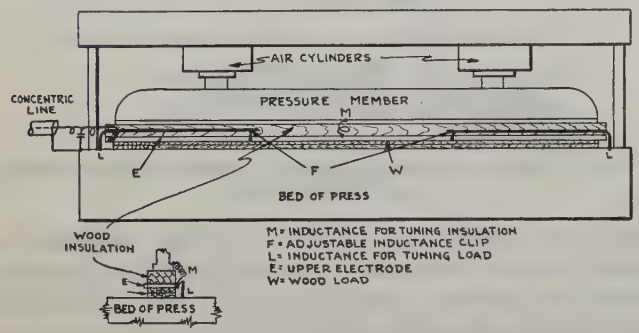

Fig. 20—Wood-gluing press adapted for radio-frequency heating at 10 megacycles.

This same press was later modified for the gluing of gusset joints in large plywood sheets. Because the large sheets had to be fed through the press, the tuning inductor arrangement of Fig. 19 was impracticable. The frequency was lowered to 10 megacycles so that only two tuning inductors would be required to give a sufficiently uniform voltage distribution. These inductors which had a value of about 0.6 microhenry were connected to the upper electrode at points three feet from the ends as shown in Fig. 20. As they were long enough to extend to the ends of the press where they were connected to the press bed, the passage of large sheets through the press was not obstructed. The voltage distribution on the press plates with and without the tuning inductors connected is shown in Fig. 21. This gluing

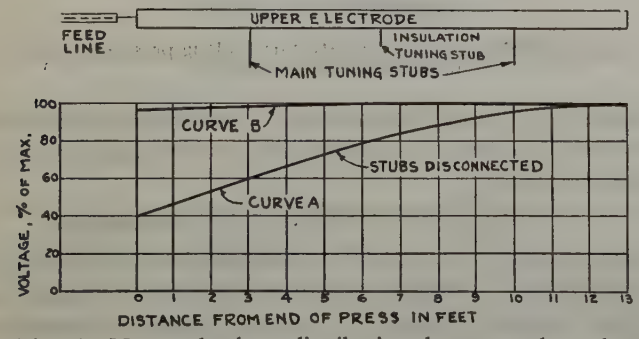

Fig. 21—Measured voltage distribution along press electrodes of Fig. 20 at 10 megacycles.

operation involved the heating of a glue line at the center of a volume of fir plywood $2\frac{1}{2}$ inches wide, 0.5 inch thick, and 13 feet long to a temperature of 240 degrees Fahrenheit and was accomplished in 1.1 minutes with a power input to the press of $7\frac{1}{2}$ kilowatts. The glue-line temperature and power input to the oscillator during the gluing cycle are shown in Fig. 22.

VI. Measurement of Temperature at the Glue Line

The glues that lend themselves most readily to a radiothermic application have a definite time-temperature relation which must be observed for reliable bonding. This makes temperature measurement at the glue line of importance during preliminary adjustments, so that a proper co-ordination between the applied power and gluing cycle may be established.

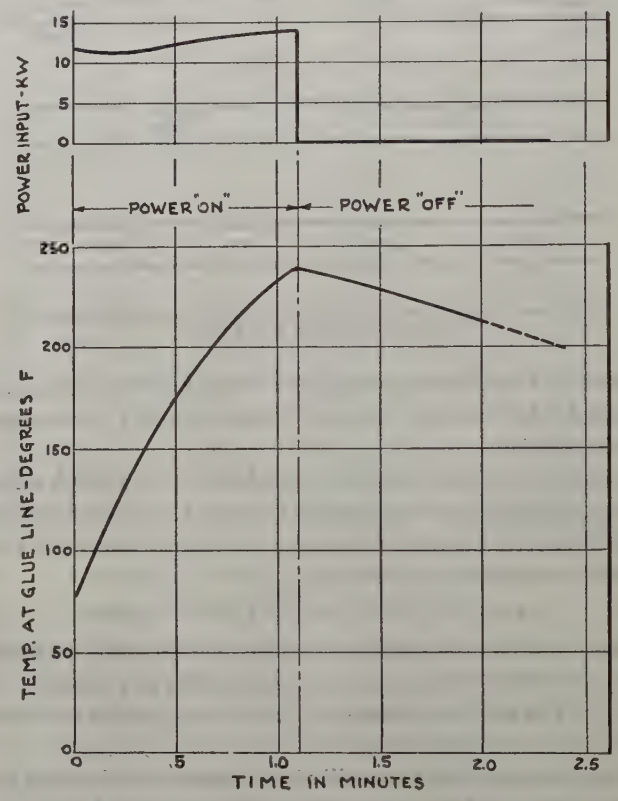

Fig. 22—Gluing cycle for $\frac{1}{4}$-inch plywood gusset joints.

The simplest means of doing this is to embed an iron-constantan couple in the glue line and permit the ends to extend slightly so that an instrument like the Leeds and Northrup temperature potentiometer may be quickly connected for a reading when the radio-frequency power is shut off. Such a plan is of small value if continuous temperature readings are desired, for with the instrument connected in this manner during the application of power, induced voltage may cause a spurious reading or may damage the potentiometer. One means of providing radio-frequency isolation of the thermocouple from the potentiometer is to wind a length of the paired wires into a choke coil of about 100 turns on a $\frac{7}{8}$-inch diameter bakelite tube. This device permits continuous readings to be made on 10- to 15-megacycle gluing operations and was used to obtain the data of Fig. 22.

Another plan makes use of a well-known property of

concentric transmission lines by which a high impedance is developed across one end of a quarter-wave line when the other end is short-circuited. If the thermocouple leads are led through the inner conductor and by-passed to it at each end as shown in Fig. 23 the connections to the potentiometer will be at ground potential while the couple at the open end of the line may be operated at any potential. It is obvious that such a method can be used for any single frequency that is high enough to permit the use of a reasonable length of line.

VII. Conclusion

An examination has been made of the various factors involved in the rapid gluing of wood by means of radio-frequency power; namely, power concentration, frequency, voltage gradient, dielectric constant, and power factor.

Nonuniform heating in large presses due to standing waves on the electrodes may be eliminated by proper adjustment of inductive tuning stubs connected across the load. The stubs may be used to tune the press to compensate for loads of varying characteristics.

An efficient means for coupling a large press to an oscillator has been developed and satisfactory glued joints may be made in a minute with catalyzed glue. A

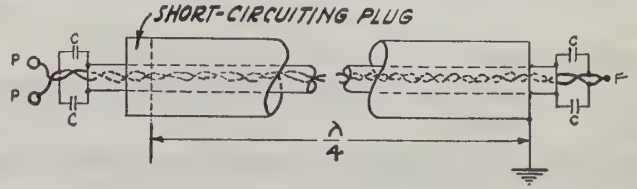

Fig. 23—Quarter-wave filter for thermocouple.

quarter-wave line or an electrically equivalent network is very helpful in reducing the effect of variations in the load on the oscillator during the gluing cycle.

It is hoped that this paper will help to emphasize the complementary advantages of radio-frequency heating and thermosetting glues for the rapid production of permanent wood bonds.

Heat-Conduction Problems in Presses Used for Gluing of Wood*

GEORGE H. BROWN†, FELLOW, I.R.E.

Summary—**The application of heat-flow equations to the heat-flow problems in wood-gluing presses is considered. By means of a transmission-line analogy, the solution of heat-flow problems with internally generated heat is greatly simplified. The general equation is set up and applied specifically to the problem of radio-frequency heating of wood, while the wood is contained between two cool plates. Comparison of this type of heating with the usual hot-plate-press method reveals important advantages for the radio-frequency method.**

The formulas developed have been illustrated by means of curves based upon the constants of wood. However, the general equations apply as well to the heating of any other insulating material, where the heating by radio frequency is done by essentially a dielectric-loss phenomena.

Introduction

HOT-PLATE presses are now used extensively in the wood-gluing industry in the manufacture of plywood, joining of sheets of plywood, the manufacture of spars from small straight-grained sections, and in the manufacture of propeller blanks. Recent work with cold-plate presses in which the wood sections are heated with radio-frequency power[1] has shown that this method has many advantages over the hot-plate methods.

It is the object of this paper to examine theoretically the heat-conduction problems encountered in both methods. In this way, the methods may be compared and also quantitative relations which are difficult to establish experimentally may be established.

The Differential Equation of Heat Conduction[2]

In either method of gluing, the wood plates or strips are placed between two large pressure plates. Heat is then applied to the wood by conduction from the heated pressure plates or the heat is generated in the wood by means of radio-frequency power. In either case, the heat-flow problem is essentially that of a single dimension, normal to the faces of the pressure plates. That is, any plane through the wood parallel to the pressure plates is a plane of equal temperature.

Under this limiting condition, we shall now examine a small rectangular box of the material. (Fig. 1.) The box has a width a, height b, and a length dx. Heat may be generated in each cubic centimeter by electrical or chemical means. Let H be the rate at which heat is generated measured in gram calories per second per cubic centimeter. H is a function of both time and distance.

* Decimal classification: R590×R536. Original manuscript received by the Institute, March 30, 1943.

† RCA Laboratories, Princeton, New Jersey.

[1] R. A. Bierwirth and C. N. Hoyler, "Radio-frequency heating applied to wood gluing," Proc. I.R.E., this issue, pp. 529–537.

[2] Analytical studies of the flow of heat in solid bodies date back to 1812 when Fourier's memoir "Theorie du mouvement de la chaleur dans les corps solides" was presented to the French Academy. That the literature pertaining to heat flow is abundant is illustrated by calling attention to the references, seven hundred and eighty-nine in number, listed by William H. McAdams, pp. 419–448, "Heat Transmission," second edition, 1942, McGraw-Hill Book Company, New York, N. Y. It was the thought in preparing this section and the one following that it might be well to include the essential derivations so that the interested reader to whom the field is new could follow the reasoning without excursions to reference books which at times may not be readily available.

Then the heat generated within the small box in a short interval of time dt is

$$Q_0 = Hab \cdot dx \cdot dt. \tag{1}$$

The temperature at point 0 is u degrees centigrade. Then the increase in heat stored in the box during the time dt is

$$Q_1 = c\rho(du/dt)d \cdot ab \cdot dx \tag{2}$$

where c = density (grams per cubic centimeter)

ρ = specific heat (calories per gram per degree centigrade).

The temperature gradient at point 0 is $(du/dx)_0$. The gradient at point B is

$$(du/dx)_B = (du/dx)_0 + [d/dx(du/dx)]_{x_0} dx/2$$

and the gradient at point A is

$$(du/dx)_A = (du/dx)_0 - [d/dx(du/dx)]_{x_0} dx/2.$$

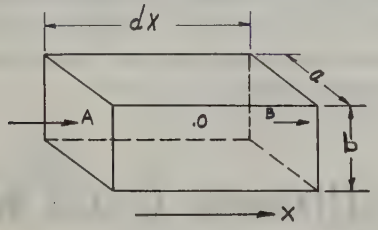

Fig. 1—Elementary box.

The heat flowing *out* through the face of the box at A in time dt is

$$Q_2 = k(du/dx)_A ab \cdot dt \tag{3}$$

where

k = thermal conductivity

$$\left(\frac{\text{calories per square centimeter per second}}{\text{degrees centigrade per centimeter}}\right).$$

The heat flowing out through the face at B is

$$Q_3 = -k(du/dx)_B ab \cdot dt. \tag{4}$$

From the conservation of energy,

$$Q_0 = Q_1 + Q_2 + Q_3. \tag{5}$$

Thus

$$Hab \cdot dx \cdot dt = c\rho(du/dt) \cdot ab \cdot dx \cdot dt + kab \cdot dt[(du/dx)_0 - \{d/dx(du/dx)\}_{x_0} dx/2] - kab \cdot dt[(du/dx)_0 + \{d/dx(du/dx)\}_{x_0} dx/2] \tag{6}$$

or[3]

$$H = c\rho(du/dt) - k(d/dx)(du/dx). \tag{7}$$

The Case of Hot-Plate Presses

In the case of ordinary hot-plate presses, no heat is generated within the wood. Then (7) becomes

$$c\rho(du/dt) = k(d^2u/dx^2). \tag{8}$$

This equation is often written as

$$du/dt = \alpha^2(d^2u/dx^2) \tag{9}$$

where $\alpha^2 = k/c\rho$ is called the "thermal diffusivity." This equation may be solved by the standard method of products. The temperature u is a function of both x and t. We assume that u may be expressed as a product of two functions X and T, where X is a function of x alone and T is a function of t alone.

$$u = X(x) \cdot T(t). \tag{10}$$

Substituting (10) in (9)

$$X \cdot (dT/dt) = \alpha^2 T(d^2X/dx^2) \tag{11}$$

or

$$\frac{dT/dt}{\alpha^2 T} = \frac{d^2X/dx^2}{X}. \tag{12}$$

Since T is a function of t alone and X is a function of x alone, and since x and t are entirely independent variables, the relation of (12) may be satisfied only if each side of (12) is equal to a constant. Then

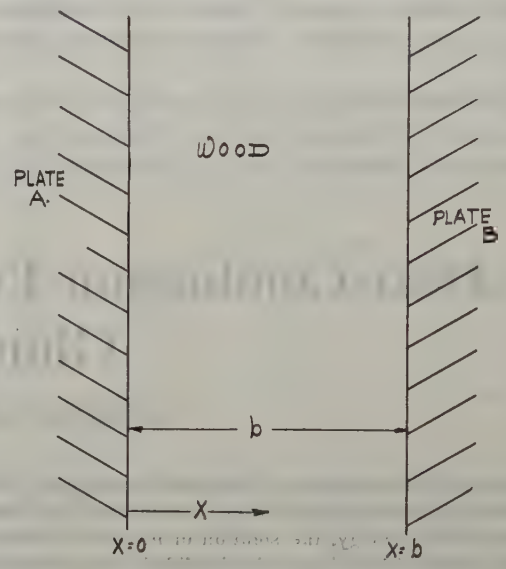

Fig. 2—Layer of wood between metal press plates.

$$\frac{dT/dt}{\alpha^2 T} = \frac{d^2X/dx^2}{X} = -m^2 \tag{13}$$

where m is an unknown and arbitrary constant. Then $X = A \cos mx + B \sin mx$ and $T = Ce^{-\alpha^2 m^2 t}$ satisfy (12), so that

$$u = e^{-\alpha^2 m^2 t}[M \cos mx + N \sin mx] \tag{14}$$

where m, M, and N must be determined by the boundary conditions.

At a time $t = 0$, the piece of wood (or a stack of wood strips) of total thickness b is placed between the two metal plates A and B shown in Fig. 2. These plates are steam or electrically heated and have a great enough thermal capacity that the plate temperature stays essentially constant.

Let β = temperature of plate A.

γ = temperature of plate B.

At the time that the wood is placed between the plates, the temperature distribution through the wood is $f(x)$, a known function. Therefore, the solution (14) must satisfy the following boundary conditions.

[3] Equation (7), which takes into account heat generated within the material, is given as equation (7-45), p. 232, Margenau and Murphy, "The Mathematics of Physics and Chemistry," D. Van Nostrand Company, Inc., New York, N. Y., 1943. However, further discussion of the solution of the equation by these authors is confined to the case where no time variation exists or to the case where no heat is generated within the material.

When $x=0$, $u=\beta$ for all values of t
$x=b$, $u=\gamma$ for all values of t
$t=0$, $u=f(x)$.
Then (14) becomes[4]

$$u = \beta + (\gamma - \beta)\left[\frac{x}{b} + \frac{2}{\pi}\sum_{n=1}^{n=\infty}\left\{\frac{(-1)^n}{n}\epsilon^{-(n^2\pi^2\alpha^2 t)/b^2}\sin\left(\frac{n\pi x}{b}\right)\right\}\right] + \frac{2}{b}\sum_{n=1}^{n=\infty}\left\{\epsilon^{-(n^2\pi^2\alpha^2 t)/b^2}\sin\left(\frac{n\pi x}{b}\right)\int_{\lambda=0}^{\lambda=b}[f(\lambda)-\beta]\sin\left(\frac{n\pi\lambda}{b}\right)d\lambda\right\}. \quad (15)$$

(a) Two Equal-Temperature Hot Plates

The most common hot-plate press has two equal-temperature plates. Then $\gamma=\beta$. Since the behavior of temperature depends on differences of temperature rather than absolute temperature, we shall assume that the wood is a constant temperature throughout at time $t=0$, so that $f(x)=f(\lambda)=a$ constant = zero. Also we will set the plate temperature at unity, so $\gamma=\beta=1$. Then (15) reduces to

$$\begin{aligned} u &= 1 + \frac{2}{b}\sum_{n=1}^{n=\infty}\left\{\epsilon^{-(n^2\pi^2\alpha^2 t)/b^2}\sin\left(\frac{n\pi x}{b}\right)\int_{\lambda=0}^{\lambda=b}(-1)\sin\left(\frac{n\pi\lambda}{b}\right)d\lambda\right\} \\ &= 1 - \frac{4}{\pi}\sum_{n=0}^{n=\infty}\frac{\epsilon^{-((2n+1)^2\pi^2\alpha^2 t)/b^2}}{(2n+1)}\sin\left(\frac{[2n+1]\pi x}{b}\right) \\ &= 1 - \frac{4}{\pi}\left[\epsilon^{-(\pi^2\alpha^2 t)/b^2}\sin\left(\frac{\pi x}{b}\right) + \frac{\epsilon^{-(9\pi^2\alpha^2 t)/b^2}\sin(3\pi x/b)}{3} + \frac{-(25\pi^2\alpha^2 t)/b^2\sin(5\pi x/b)}{5} + \cdots\right]. \end{aligned} \quad (16)$$

Average values for the constants of wood are
ρ = specific heat = 0.65 calorie per gram per degree centigrade
c = density = 0.53 gram per cubic centimeter
k = 0.00036 calorie per square centimeter per second for 1 degree centigrade per centimeter temperature gradient
$\alpha^2 = k/c\rho = 0.00104$.
Then (16) becomes

$$u = 1 - \frac{4}{\pi}\left[\epsilon^{-(0.01025t)/b^2}\sin\left(\frac{\pi x}{b}\right) + \frac{\epsilon^{-(0.0923t)/b^2}\sin(3\pi x/b)}{3} + \frac{\epsilon^{-(0.2562t)/b^2}\sin(5\pi x/b)}{5} + \cdots\right]. \quad (17)$$

In (17), t is expressed in seconds and b is in centimeters, while u is in degrees centigrade. Fig. 3 shows temperature of the wood at a number of points through the wood as a function of time. The abscissa scale is given on the basis of a number of wood thicknesses. This is possible since it is seen from (17) that the time required to reach a certain value of u is proportional to the square of the thickness b.

Fig. 4 shows temperature distribution through the wood for a number of time intervals.

While Fig. 3 is very general and contains all the necessary information for a study of the problem, it seems desirable to use it more specifically. The ordinates of Fig. 3 represent wood temperature where the plate temperature is 1 degree centigrade and where the initial wood temperature is 0 degree centigrade. These ordinates also may be regarded as a fraction of the temperature *interval* between the initial wood temperature and the plate temperature. Then, in any general case,

$$u_w = (u_p - u_0)u + u_0 \quad (18)$$

where u_p = plate temperature
u_0 = initial wood temperature
u = specific wood temperature from Figs. 3 and 4
u_w = wood temperature as a function of time.

It is apparent that as long as u is the fraction taken from Figs. 3 and 4, all other temperatures in (18) may be expressed in the centigrade or the Fahrenheit scale.

Fig. 5 shows the temperature at the center of the wood for a number of plate temperatures, where the initial wood temperature was 80 degrees Fahrenheit. The broken line marks 240 degrees Fahrenheit as the necessary wood temperature for catalyzed gluing.[1] The highest plate temperature used was 400 degrees Fahrenheit since it has been experimentally determined that this is the scorch temperature of the ordinary woods used in the glue industry.

Fig. 6 was constructed from Fig. 5 and shows the plate temperature required to raise the wood from 80 to 240 degrees Fahrenheit in a time interval shown by the abscissa.

Fig. 7 shows Fig. 4 converted on the basis where the plate temperature was chosen to bring center of the wood to 240 degrees Fahrenheit in the time interval shown. The top curve is, of course, an absurd case since the plate temperature of 1145 degrees Fahrenheit would set the wood on fire immediately. It is included,

[4] W. E. Byerly, "Fourier's Series and Spherical, Cylindrical, and Ellipsoidal Harmonics," Ginn and Co., New York, N. Y., 1893, page 108, example 1.

however, to illustrate the extreme temperature distribution encountered in hot-plate gluing.

The heat energy supplied to a box of the wood which is 1 square centimeter in cross section and b centimeters

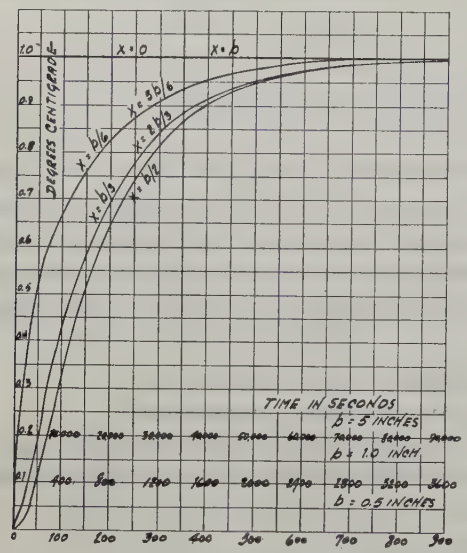

Fig. 3—Specific wood temperature as a function of time, at various points in the wood, when two equal-temperature hot plates are used.

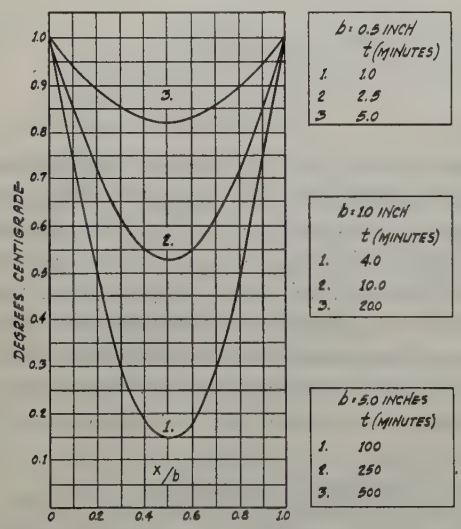

Fig. 4—Specific wood temperature distribution throughout the wood, with two equal-temperature hot plates.

thick, in the time interval between time of application of the plates and any time t is

$$\text{calories} = c\rho\left[\int_{x=0}^{x=b} u(x)dx\right]. \quad (19)$$

Substituting (16) in (19),

$$\text{calories} = c\rho b\left[1-8/\pi^2\left\{\left(\epsilon^{-(\pi^2\alpha^2t)/b^2}\right)+\left(\epsilon^{-(9\pi^2\alpha^2t)/b^2}\right)/9 + \left(\epsilon^{-(25\pi^2\alpha^2t)/b^2}\right)/25+\cdots\right\}\right] \quad (20)$$

or watt-seconds per square centimeter of surface

$$= 4.187(u_p-u_0)c\rho b\left[1-8/\pi^2\left\{\epsilon^{-(\pi^2\alpha^2t)/b^2}+\left(\epsilon^{-(9\pi^2\alpha^2t)/b^2}\right)/9 + \left(\epsilon^{-(25\pi^2\alpha^2t)/b^2}\right)/25+\cdots\right]\right. \quad (21)$$

(u_p and u_0 in degrees centigrade).

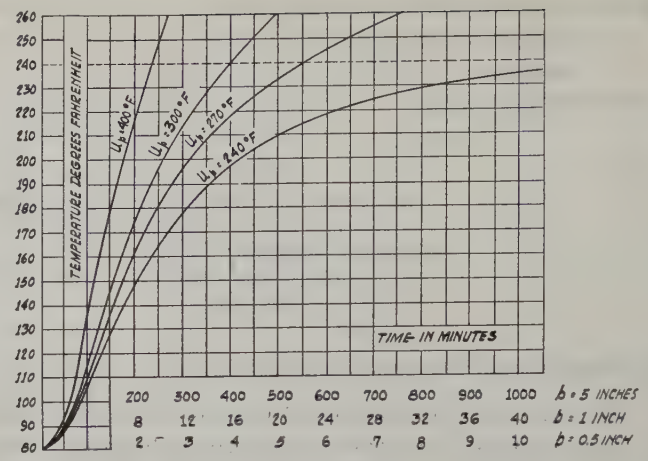

Fig. 5—Temperature at center of wood for a number of plate temperatures. Initial wood temperature was 80 degrees Fahrenheit.

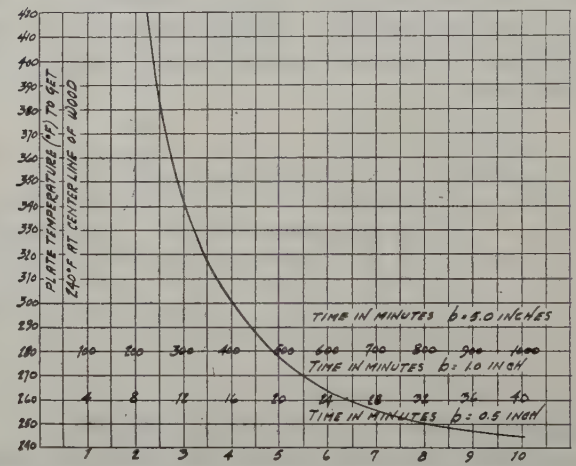
PLATE TEMPERATURE (U_p) TO GET 240°F AT CENTER LINE OF WOOD
TIME IN MINUTES b = 5.0 INCHES
TIME IN MINUTES b = 1.0 INCH
TIME IN MINUTES b = 0.5 INCH

Fig. 6—Plate temperature required to raise the wood center line to 240 degrees Fahrenheit in the time interval shown by the abscissa. Initial wood temperature was 80 degrees Fahrenheit.

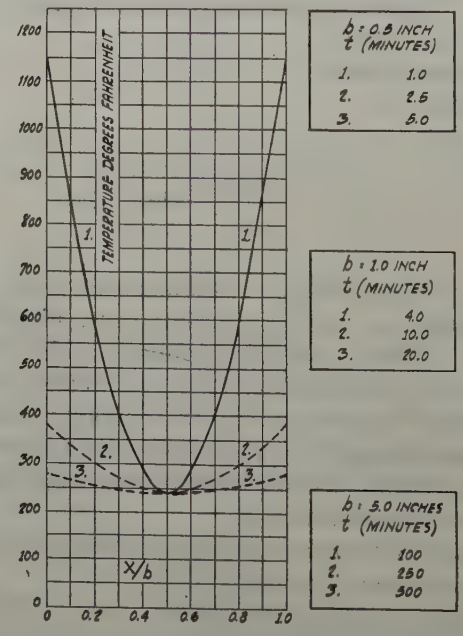

Fig. 7—This diagram, constructed from Fig. 4 and equation (18), shows temperature distribution through the wood for a number of plate temperatures chosen to bring the center line to 240 degrees Fahrenheit in the time interval chosen.

Using (21) and the times and plate temperatures of Fig. 6, Fig. 8 was prepared.

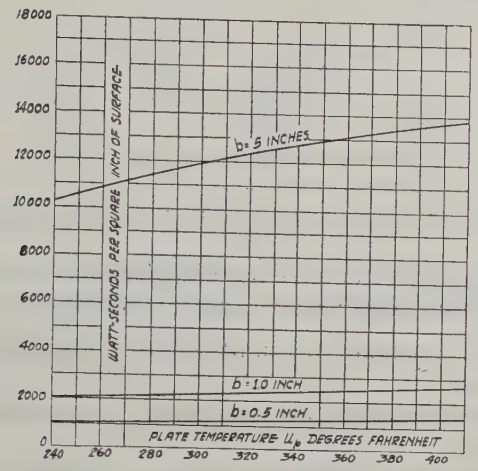

Fig. 8—Energy absorbed by wood versus plate temperature. The time is that required to raise the center line of the wood to 240 degrees Fahrenheit.

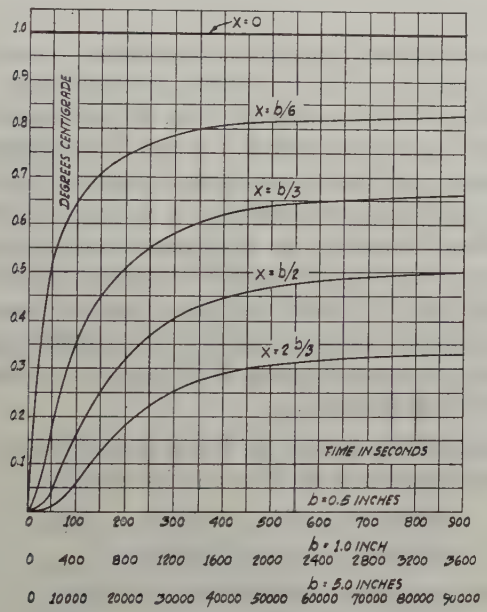

Fig. 9—Specific wood temperature as a function of time, when press uses one hot plate and one cold plate.

(*b*) *One Hot Plate and One Cold Plate*

A number of hot-plate presses use only one hot plate, with the other plate, which is generally the press bed, remaining at room temperature. At the time that the wood is placed in the press, it is also at room temperature. In Fig. 2, plate A is then at 1 degree centigrade, while plate B is at zero degrees centigrade. In (15), $\beta=1$, $\gamma=0$, and $f(\lambda)=0$. Then (15) becomes

$$u = 1 - (x/b) - (2/\pi)\left[\epsilon^{-(\pi^2\alpha^2t)/b^2}\sin(\pi x/b) + (\epsilon^{-(4\pi^2\alpha^2t)/b^2})/2\sin(2\pi x/b) + (\epsilon^{-(9\pi^2\alpha^2t)/b^2})/3\sin(3\pi x/b) + (\epsilon^{-(16\pi^2\alpha^2t)/b^2})/4\sin(4\pi x/b) + \cdots\right]. \quad (22)$$

Fig. 9 shows the temperature as a function of time for a number of points in the wood, while Fig. 10 shows temperature distribution through the wood.

If the temperature of the wood at starting point is 80 degrees Fahrenheit, and plate B is at 80 degrees Fahrenheit, Figs. 11 and 12 apply for plate A at a temperature of 400 degrees Fahrenheit. These figures were constructed from Figs. 9 and 10 by applying (18).

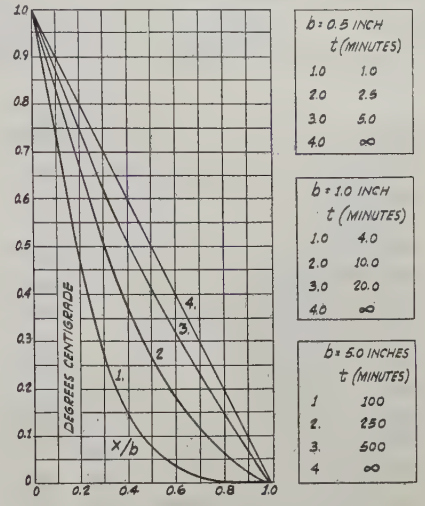

Fig. 10—Specific wood temperature distribution through the wood, with one hot and one cold plate.

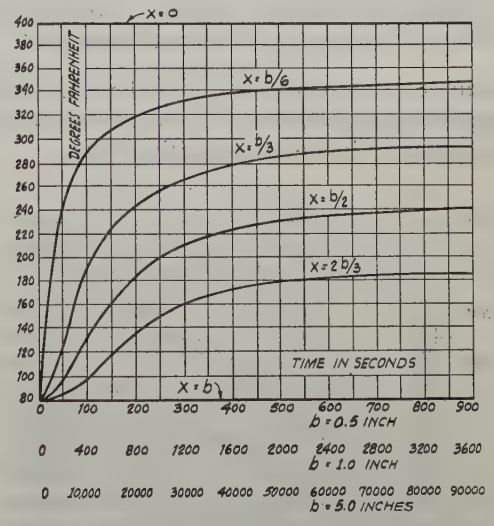

Fig. 11—Temperature of wood versus time. Hot plate at 400 degrees Fahrenheit. Cold plate at 80 degrees Fahrenheit. Initial wood temperature 80 degrees Fahrenheit.

In this case, the heat energy supplied by the hot plate cannot be reckoned from (19) because some heat is lost into the cold plate B. However, another method is just as simple:

$$\text{calories supplied} = \int_{t=0}^{t=t}\left[-k(du/dx)\right]_{x=0}dt. \quad (23)$$

Substituting (22) in (23), we find

$$\text{watt-seconds per square centimeter} = 4.187(u_p-u_0)b\left[(kt/b^2)+(2c\rho/\pi^2)\left\{(1-\epsilon^{-(\pi^2\alpha^2t)/b^2}) + (1-\epsilon^{-(2^2\pi^2\alpha^2t)/b^2})/2^2 + (1-\epsilon^{-(2\pi^2\alpha^2t)/b^2})/3^2 + \cdots\right\}\right]. \quad (24)$$

Since

$$2/\pi^2\left\{1 + 1/2^2 + 1/3^2 + \cdots\right\} = 2/\pi^2\cdot\pi^2/6 = 1/3$$

equation (24) becomes

watt-seconds per square centimeter

$$= 4.187(u_p - u_0)b\left[kt/b^2 + c\rho/3 - 2c\rho/\pi^2\left\{\epsilon^{-(\pi^2\alpha^2 t)/b^2} + (\epsilon^{-(2^2\pi^2\alpha^2 t)/b^2})/2 + (\epsilon^{-(3^2\pi^2\alpha^2 t)/b^2})/3^2 + \cdots\right\}\right]. \quad (25)$$

Fig. 13 was constructed from (25). The solid curve shows the total energy (joules or watt-seconds) supplied by the hot plate from starting time. The broken curve

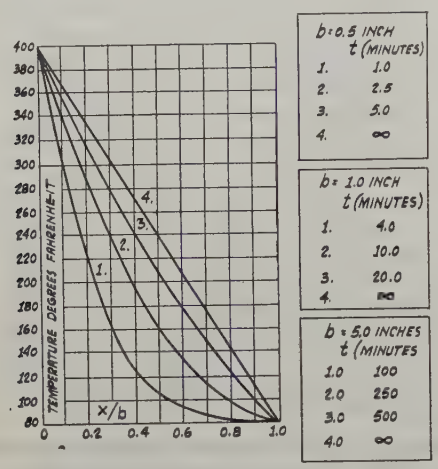

Fig. 12—Temperature of wood versus distance into the wood. Hot plate at 400 degrees Fahrenheit. Cold plate at 80 degrees Fahrenheit. Initial wood temperature 80 degrees Fahrenheit.

A is the temperature at the center of the wood. Curve B is the temperature in the wood if the same energy were released uniformly in the wood with no loss.

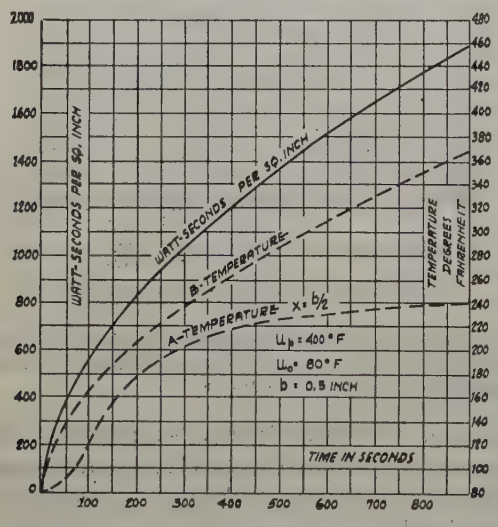

Fig. 13—Upper solid curve shows total energy supplied by the hot plate from starting time. Curve A shows temperature at the center of the wood, while curve B is the temperature of the wood if the same energy were released uniformly in the wood with no loss.

Solution of Heat-Flow Equations When Heat Is Generated within the Wood

It is well to restate that we are only considering heat flow in one dimension. Then, with energy generated within the wood, (7) holds. As stated previously, H may be a function of distance and time. The solution of (7) will be made by showing the direct analogy between the heat-flow problem and a particular transmission-line problem.

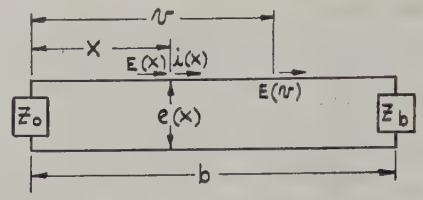

Fig. 14—Transmission line used in the heat-flow analogy.

In Fig. 14, we have a transmission line of length b. The uniformly distributed constants of the line are

R = series resistance per unit length (ohms per centimeter)

L = series inductance per unit length (henries per centimeter)

G = shunt conductance per unit length (mhos per centimeter)

C = shunt capacitance per unit length (farads per centimeter)

At one end, $x=0$, the line is terminated in an impedance Z_0 while at $x=b$ the line is terminated in an impedance Z_b. In series with the line, we have an induced voltage set up by an external force. This voltage may be set up by an impinging electromagnetic field or by inserting a battery or generator in series with the line. This voltage intensity is $E(v)$ measured in volts per centimeter. We wish to find the current and voltage along the line at a point x units from the impedance Z_0 at any instant of time, under the condition that $E(v)$ is induced all along the line. $E(v)$ may be a function of both time and distance.

Let e = the voltage across the line x units from Z_0

i = the current in the line x units from Z_0.

Then $$de/dx = -Ri - L(di/dt) + E(x) \quad (26)$$

and $$di/dx = -Ge - C(de/dt) \quad (27)$$

or, making use of the differential operator p,

$$de/dx = -(R + pL)i + E(x) \quad (28)$$

and $$di/dx = -(G + pC)e. \quad (29)$$

From (29) $$e = [(-1)/(G + pC)](di/dx) \quad (30)$$

and $$de/dx = [(-1)/(G + pC)](d^2i/dx^2). \quad (31)$$

Substituting (31) in (28),

$$E(x) = (R + pL)i - (1/G + pC)(d^2i/dx^2) \quad (32)$$

The solution for the current i at a point x is

$$i(x) = \left\{[Z_c^2 + Z_bZ_0]\int_{v=x}^{v=b} E(v)\cosh\gamma(b+x-v)dv + [Z_c^2 - Z_bZ_0]\int_{v=0}^{v=b} E(v)\cosh\gamma(b-x-v)dv + [Z_c^2 + Z_bZ_0]\int_{v=0}^{v=x} E(v)\cosh\gamma(b-x+v)dv + [Z_cZ_b + Z_cZ_0]\int_{v=x}^{v=b} E(v)\sinh\gamma(b+x-v)dv\right.$$

$$+[Z_aZ_b-Z_cZ_0]\int_{v=0}^{v=b}E(v)\sinh\gamma(b-x-v)dv$$

$$+[Z_aZ_b-Z_cZ_0]\int_{v=0}^{v=x}E(v)\sinh\gamma(b-x+v)dv$$

$$\div\{2Z_c\sinh(\gamma b)[Z_c^2-Z_0Z_b+(Z_0+Z_b)Z_c\coth(\gamma b)]\} \quad (33)$$

where $$Z_c=\sqrt{(R+pL)/(G+pC)} \quad (34)$$

and $$\gamma^2=(R+pL)(G+pC). \quad (35)$$

We next propose that the resistance per unit length R and the shunt capacitance per unit length C be made zero. Then the transmission line has only the constants of series inductance L and shunt leakage G. Under this condition, (32) becomes

$$E(x)=pLi-(1/G)(d^2i/dx^2)=L(di/dt)-(1/G)(d^2i/dx^2) \quad (36)$$

and (30) is

$$e=-(1/G)(di/dx). \quad (37)$$

Comparing (36) with (7), we see that if $i(x)$ is analogous to the temperature u

$$L=c\rho \quad (38)$$

$$1/G=k \quad (39)$$

$$E(x)=H \text{ (gram calories per second per cubic centimeter)} \quad (40)$$

and from (37)

$$e=-(1/G)(di/dx)=-k(du/dx) \quad (41)$$

so that the voltage on the line is analogous to the heat flow across a surface (proportional to temperature gradient).[5]

Also $$Z_c=\sqrt{pL/G}=\sqrt{pc\rho k} \quad (42)$$

and $$\gamma^2=pLG=pc\rho/k=p/\alpha^2. \quad (43)$$

[5] In his book, "Heaviside's Operational Calculus," E. J. Berg points out in chapter XXVII the analogy between the heat-flow equations and the equations of a transmission line. He chooses to set temperature equal to voltage, and thus uses a transmission line which is made up of series resistance and shunt capacitance. This method is very useful for solving the ordinary heat-flow problems where hot plates are applied to a boundary, for then the constant temperature of the boundary is represented by a constant voltage at this point. In the case of internally generated heat, it seems more reasonable to use current in relation to temperature, with a transmission line made up of series inductance and shunt leakage. In this way, one avoids the notion of constant current admitted to the line at a number of points in the line. It is evident that problems may be worked by either choice. After rather careful consideration, the writer decided on the temperature-current identity because of the nature of the problems to be considered.

In the case of the wood-gluing press, where the heat is generated by means of radio frequency, we shall assume that the plates remain at zero temperature. Then $i_{x=0}=0$ at all times, and $i_{x=b}=0$ at all times. This is accomplished by open-circuiting the line at both ends, that is, $Z_b=Z_0=\infty$. Also, we shall assume that the heat is generated uniformly throughout the wood, so that $H=E(x)$ is a constant. Under all these conditions, (33) becomes

$$i(x)=u(x)=\frac{E}{2Z_c\sinh(\gamma b)}\left\{\int_{v=x}^{v=b}\cosh\gamma(b+x-v)dv-\int_{v=0}^{v=b}\cosh\gamma(b-x-v)dv+\int_{v=0}^{v=x}\cosh\gamma(b-x+v)dv\right\} \quad (44)$$

or $$u(x)=\frac{E}{\gamma Z_c}\left[\frac{\sinh(\gamma b)-\sinh(\gamma x)-\sinh\gamma(b-x)}{\sinh(\gamma b)}\right]. \quad (45)$$

Substituting (42) and (43) in (45),

$$u(x)=\frac{E}{pL}\left[\frac{\sinh(\sqrt{pLG}\,b)-\sinh(\sqrt{pLG}\,x)-\sinh(\sqrt{pLG}\,[b-x])}{\sinh(\sqrt{pLG}\,b)}\right]. \quad (46)$$

The operational solution of (46) is [see Appendix I, equation (75)]

$$u(x)=\frac{4EGb^2}{\pi^3}\sum_{s=1}^{s=\infty}\frac{\sin[(2s-1)(\pi x/b)]}{(2s-1)^3}\left[1-\epsilon^{-((2s-1)^2\pi^2t)/b^2LG}\right]. \quad (47)$$

Substituting (38) and (39) in (47)

$$u(x)=\frac{4Hb^2}{\pi^3k}\sum_{s=1}^{s=\infty}\frac{\sin[(2s-1)(\pi x/b)]}{(2s-1)^3}\left[1-\epsilon^{-((2s-1)^2\pi^2\alpha^2t)/b^2}\right]$$

$$=\frac{4Hb^2}{\pi^3k}\left[(1-\epsilon^{-(\pi^2\alpha^2t)/b^2})\sin\left(\frac{\pi x}{b}\right)+\frac{(1-\epsilon^{-(3^2\pi^2\alpha^2t)/b^2})}{3^3}\sin\left(\frac{3\pi x}{b}\right)+\frac{(1-\epsilon^{-(5^2\pi^2\alpha^2t)/b^2})}{5^3}\sin\left(\frac{5\pi x}{b}\right)+\cdots\right]. \quad (48)$$

This equation reveals a number of interesting facts concerning radio-frequency heating, a few of which we shall now examine. In the following calculations we will use $\alpha^2=k/c\rho=0.00104$, the same value as used in the hot-press calculations.

Figs. 15 and 16 were constructed from (48) to show the temperature distribution through the wood for a number of heating times. The temperature scale is purely relative.

We are generally interested in the temperature at the center of the wood. Under this condition, $x=b/2$ and (48) becomes

$$u\left(x=\frac{b}{2}\right)=\frac{4Hb^2}{\pi^3k}\left[\left(1-\epsilon^{-(\pi^2\alpha^2t)/b^2}-\frac{(1-\epsilon^{-(3^2\pi^2\alpha^2t)/b^2})}{3^3}+\frac{(1-\epsilon^{-(5^2\pi^2\alpha^2t)/b^2})}{5^3}\cdots\right)\right]. \quad (49)$$

Fig. 17 shows the temperature rise at the center of the wood as a function of time for a wood thickness of 0.5 inch for a number of rates of energy generation.

(Watts = 4.187 × gram calories per second.[6]) Here it is assumed that the initial wood temperature is 80 degrees Fahrenheit and that the plates remain at 80 degrees Fahrenheit.

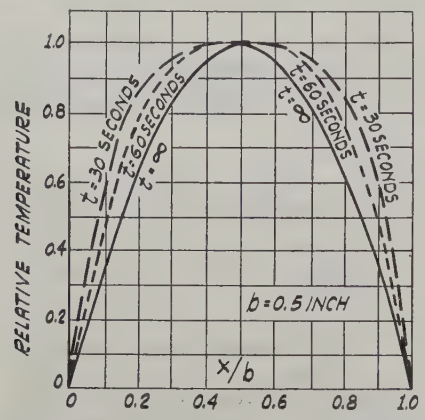

Fig. 15—Specific wood temperature distribution through the wood. Thickness of wood is one-half inch.

In Fig. 17, we see that for low power concentrations the wood temperature does not approach the necessary glue-setting temperature of 240 degrees Fahrenheit. It is interesting to examine (49) to establish the limits of temperature. As $t \doteq \infty$

$$u\left(x=\frac{b}{2},\ t \doteq \infty\right)=\frac{4Hb^2}{\pi^3 k}\left[1-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\cdots\right]$$
$$=\frac{4Hb^2}{\pi^3 k}\left[\frac{\pi^3}{32}\right]=\frac{Hb^2}{8k}. \qquad (50)$$

This formula gives us Fig. 18, which shows the mini-

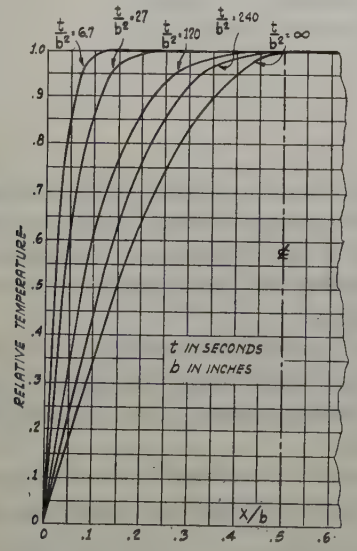

Fig. 16— Specific temperature distribution curves generalized with respect to time and wood thickness.

mum power concentration necessary to achieve a temperature *increase* of 160 degrees Fahrenheit at the center of the wood.

Now, if there were no loss of heat at the pressure plates, the temperature increment would be

[6] 4.187 watts = 1 gram calorie per second.

$$u_{\text{no losses}} = Ht/c\rho. \qquad (51)$$

So, from (49) and (51), the ratio of the temperature increments with plate losses and with no losses for equal power densities is $(k/c\rho = \alpha^2)$

$$\frac{\Delta u_{\text{losses}}}{\Delta u_{\text{no losses}}}=\frac{4b^2}{\pi^3\alpha^2 t}\left[(1-\epsilon^{-(\pi^2\alpha^2 t)/b^2})-\frac{(1-\epsilon^{-(3^2\pi^2\alpha^2 t)/b^2})}{3^3}\right.$$
$$\left.+\frac{(1-\epsilon^{-(5^2\pi^2\alpha^2 t)/b^2})}{5^3}\cdots\right] \qquad (52)$$

and

$$\left(\frac{\Delta u_{\text{losses}}}{\Delta u_{\text{no losses}}}\right)_{t\doteq\infty}=\frac{b^2}{8\alpha^2 t}. \qquad (53)$$

Likewise, the power densities required to raise the wood through the same temperature increment in a given time is

$$H_{\text{losses}}/H_{\text{no losses}} = 1/\text{equation (52)}. \qquad (54)$$

Fig. 19 shows the ratio of temperatures for equal power densities, and the ratio power densities required

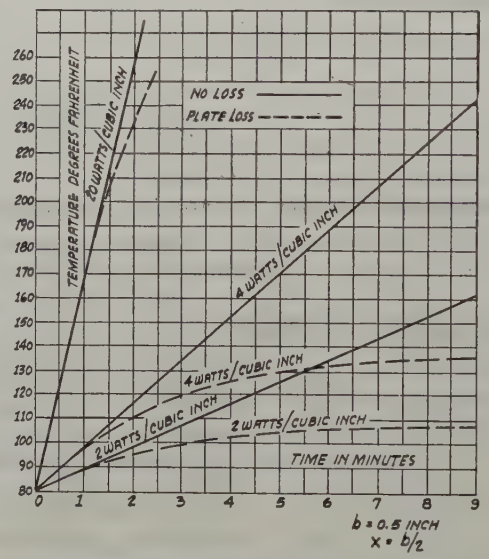

Fig. 17—Temperature rise at center of wood for a number of rates of energy generation.

to raise the wood through the same temperature increment in a given time. Note that the time scales vary as the square of the wood thickness.

Fig. 19 shows that economy in energy is attained by using short heating times. Fig. 20 shows the time allowable for heating various wood thicknesses where the power (and energy) does not exceed that used with no plate losses by more than 10 per cent.

Fig. 21 shows the time taken to achieve a given temperature increment as a function of the power density when no heat is lost in the press plates. Figs. 21 and 19 may be used for estimating purposes in planning a gluing installation.

By way of example, Bierwirth and Hoyler[1] found that 28 kilowatts applied to a press raised the temperature of a set of spruce boards to at least 260 degrees Fahrenheit in two minutes. The boards were each 13 feet long, 6 inches wide, and 5/16 inch thick. Three boards were stacked, making a total volume of $13 \times 12 \times 6 \times 15/16 = 878$ *cubic inches*, and $b = 15/16$ inch.

Assuming a starting temperature of 80 degrees Fahrenheit, we need an increment of 180 degrees Fahrenheit. From Fig. 21, for this increment and a time of two minutes, the power must be 20 watts per cubic inch with no losses. We next turn to Fig. 19. Two minutes with $b=15/16$ inch corresponds to $2/(15/16)^2=2.275$

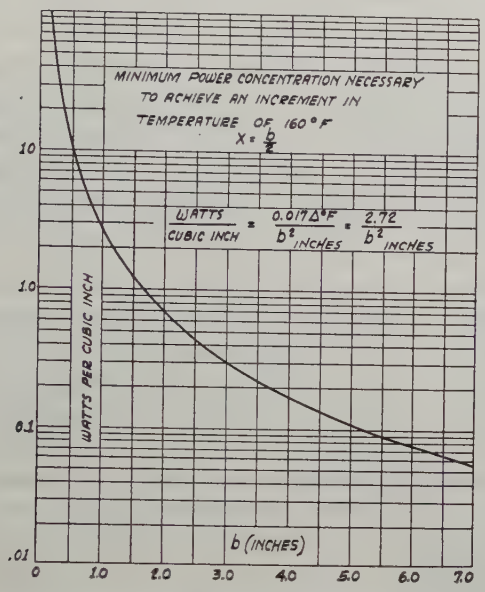

Fig. 18—Minimum power concentration necessary to achieve a temperature increment of 160 degrees Fahrenheit.

minutes on the $b=1.0$ inch scale. Then the power ratio is 1.02, and the power density required is $1.02\times20=20.4$ watts per cubic inch or $20.4\times878=17{,}900$ watts is the total power requirement. Comparing this figure with the

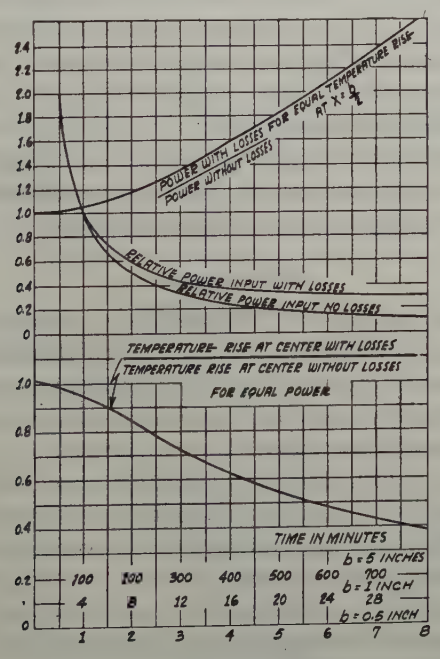

Fig. 19—Power ratio and temperature-increment ratio curves versus time.

28,000 watts fed into the press and coupling system, the efficiency is 64.0 per cent. This low efficiency is not astonishing, since an insulating wood plank two inches thick is in parallel with the sections being glued. This loss might be eliminated by using a low-loss insulator in place of the plank.

Another application was that of gluing plywood gusset joints, in which the total thickness was $b=0.5$ inch

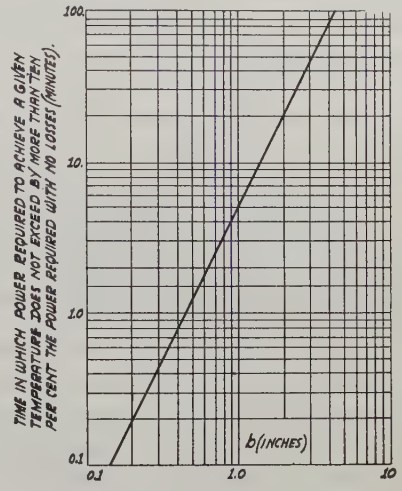

Fig. 20—Time allowable for heating when the power does not exceed by more than 10 per cent that required with no heat losses.

with a width of 2.5 inches and a length of 13 feet. The total volume was then 195 cubic inches. For a gluing time of 1.1 minutes and a temperature rise of 160 degrees Fahrenheit, Fig. 21 shows a power of 32 watts per cubic inch with no heat loss. Fig. 19 shows a correction factor of 1.07 so that the total power requirements are

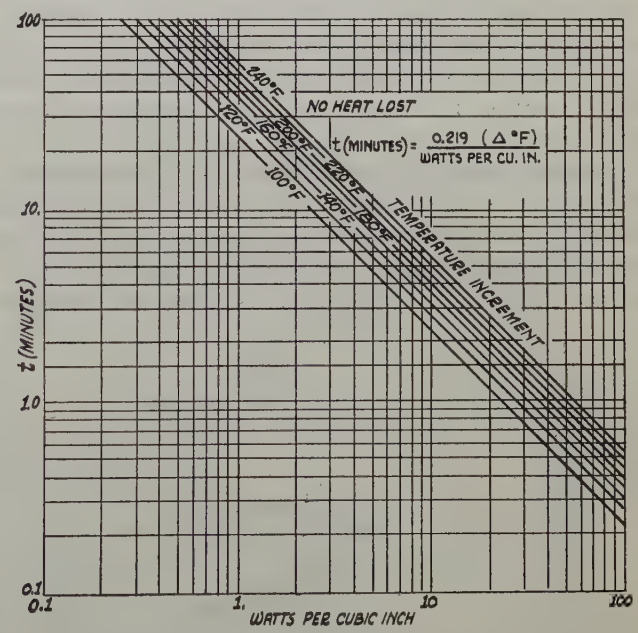

Fig. 21—Time required to achieve a given temperature increment versus power density. No heat is lost in the press plates.

$1.07\times32\times195=6670$ watts. Bierwirth and Hoyler report that the total input to the press was 7500 watts.

Another application of interest is that of a blank 6 feet long, 20 inches wide, and 6 inches thick, representing a total volume of 8650 cubic inches, with $b=6$ inches. Suppose that we have a power into the press of 50,000 watts and a press efficiency of 75 per cent. Then the power

density is 4.33 watts per cubic inch. From Fig. 21, this corresponds to 8.2 minutes for a temperature rise of 160 degrees Fahrenheit with no heat loss. Turning to Fig. 19, and noting that $b=6$ inches and $t=8.2$ minutes corresponds to $8.2/6^2=0.228$ minutes on the $b=1.0$ inch scale, we find the loss factor to be unity, so we might expect the 50,000-watt oscillator to do the job in 8.2 minutes.

Table I summarizes the above results, and by referring to Fig. 6, compares the radio-frequency method with the best heating time obtained by a hot-plate press using the plates at a scorching temperature of 400 degrees Fahrenheit. It is interesting to note from Fig. 19 that if a certain power gives 1-minute gluing on a $\frac{1}{2}$-inch board, reducing the power to one third of its original value extends the required gluing time to 7 minutes.

TABLE I

	Radio-Frequency Method			Hot-Plate Method			
	Power into Wood (Watts)	Gluing Time (Min.)	Energy (Watt-min.)	Gluing Time (Min.)	Energy (Watt-min.)	Hot-Plate Time R-F Time	Hot-Plate Energy R-F Energy
Gusset Joints 13′×2.5″×0.5″ (b=0.5″)	6,670	1.1	7,350	2.375	8,520	2.155	1.16
Spar 13′×6″×15/16″ (b=15/16″)	17,900	2.0	35,800	8.32	40,700	4.16	1.138
Blank 6′×20″×6″ (b=6″)	37,500	8.2	307,500	342.	409,000	41.7	1.33

Another calculation of interest is that of the transient temperature state, where heat is generated within the wood from time $t=0$ to time $t=t_1$. At this point, the heat generation is stopped and the temperature goes into a decline. We may treat this case mathematically by means of the superposition principle. The expression for temperature at the center of the wood on the basis of heat generated throughout the wood is given by (49). This is also curve A in Fig. 22. This heat continues to be generated for all time. However, at time $t=t_1$ and beyond, a negative generation of heat is imagined which just cancels the positive generation. The temperature resulting from this negative generation is

$$u_B = -\frac{4Hb^2}{\pi^3 k}\left[(1-\epsilon^{-(\pi^2\alpha^2(t-t_1))/b^2}) - \frac{(1-\epsilon^{-(3^2\pi^2\alpha^2(t-t_1))/b^2})}{3^3}+\cdots\right] \quad (55)$$

and is given by curve B, Fig. 22. Note that curve B is simply the negative of curve A, displaced to the right an amount equal to t_1. Then for time greater than t_1, the resultant temperature curve C is the algebraic sum of A and B. Once curve A has been calculated, it is a simple matter to construct curve C very quickly. In this manner, Fig. 23 was constructed, with $b=0.5$ inch, and a power density of 34.2 watts per cubic inch. The measured curve on this figure is taken from Fig. 22 of the paper by Bierwirth and Hoyler.[1]

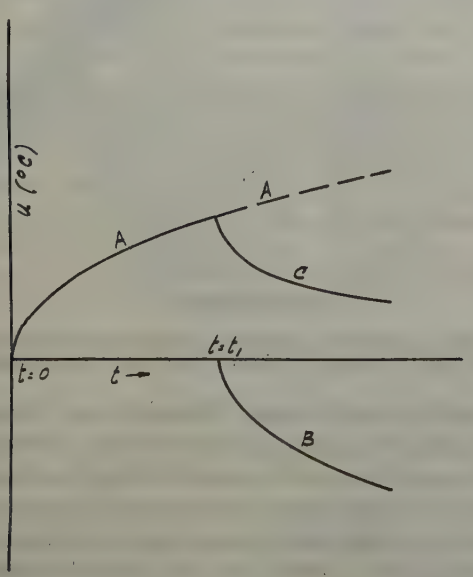

Fig. 22—Transient calculation by superposition method.

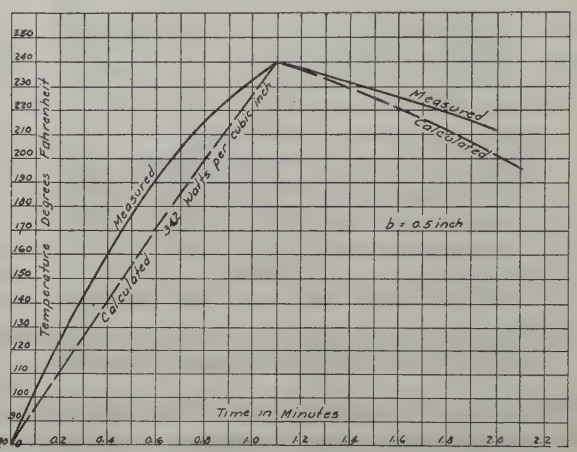

Fig. 23—Transient temperature curve with wood thickness equal to one-half inch.

This method of transient calculation is of real importance in a treatment of the self-quenching of steels when case-hardening by radio frequency.

Conclusion

The problems of heat conduction in presses used for the gluing of wood have been examined theoretically.

In a hot-plate press, where only one plate is heated, the results are so poor that this method is of no importance compared to radio-frequency heating.

Where two hot plates are used, the speed of gluing is determined by the maximum usable plate temperature, that is, the scorch temperature of the wood. It is shown that radio-frequency heating is faster. As the wood section increases in thickness, the argument in favor of radio-frequency heating becomes stronger.

In radio-frequency heating, the central section of the wood is the hottest, while in hot-plate gluing, the central section is the coolest. Thus if a temperature of say 240 degrees Fahrenheit is needed for gluing in the center of the wood, the remainder of the wood will be at a temperature less than 240 degrees, while in two-hot-plate

gluing, the remainder of the wood will be at a temperature greater than 240 degrees. Obviously, with lower surface temperatures and shorter heating times, the surface of the wood will not be dried or subjected to surface strains in radio-frequency heating to the extent that occurs in hot-plate heating.

It is perhaps evident to the reader that the analysis presented in this paper is not limited to the heating of wood. The general equations of heat flow developed in this paper may be of interest in other heating problems, such as the heating or curing of plastics.[7] In a curing problem, it may be desirable and even necessary to have a more uniform distribution of temperature than is indicated in Figs. 15 and 16. Heating of the press plates by steam or internal electrical heaters to prevent cooling at the surfaces of the plastic is desirable. By using press plates which are held at the final desired temperature and adjusting the total heating time by controlling the radio-frequency oscillator power, one may arrive at a final temperature distribution that is essentially a constant throughout the thickness of the material. Many plastic presses are equipped with water-cooled dies for cooling the material after curing. Since these dies must be heated for each pressure operation, it is possible to heat the dies at a uniform rate while the radio-frequency power is applied to the dielectric material. If the die-heating rate is controlled so that at each instant during the heating period the die is at the same temperature as the dielectric material, no heat will flow across the interface between the dielectric and the metal. Then because of the uniform nature of the generation of heat within the dielectric, the temperature throughout the dielectric will be indeed a constant.

The equations of heat flow, where the heat is generated within the object being heated, are shown to be exactly analogous to the equations of a transmission line with distributed voltage. The method of treatment is given in detail and will be used in a later paper as the basis for solving similar heat-conduction problems in metals.

Appendix I

The Operational Solution of Equation (46)

The operational equation (46) is

$$u(x) = \frac{E}{pL}\left[\frac{\sinh(\sqrt{pLG}\,b) - \sinh(\sqrt{pLG}\,x) - \sinh(\sqrt{pLG}\,[b-x])}{\sinh(\sqrt{pLG}\,b)}\right]$$
$$= \frac{E}{pL}\left[1 - \frac{\sinh(\sqrt{pLG}\,x)}{\sinh(\sqrt{pLG}\,b)} - \frac{\sinh(\sqrt{pLG}\,[b-x])}{\sinh(\sqrt{pLG}\,b)}\right]. \quad (56)$$

We shall first solve the operational relation, $[-\sinh(\sqrt{pLG}\,x)]/[\sinh(\sqrt{pLG}\,b)]$ by means of the Heaviside Expansion Theorem.

Let

$$y = -\frac{\sinh(\sqrt{pLG}\,x)}{\sinh(\sqrt{pLG}\,b)} = -\frac{1}{\left[\frac{\sinh(\sqrt{pLG}\,b)}{\sinh(\sqrt{pLG}\,x)}\right]} = \frac{Y(p)}{Z(p)} \quad (57)$$

where $Y(p)$ is a function of the operator p, and $Z(p)$ is a function of the same operator.

From (57) $$Y(p) = -1 \quad (58)$$

and $$Z(p) = \frac{\sinh(\sqrt{pLG}\,b)}{\sinh(\sqrt{pLG}\,x)}. \quad (59)$$

The Expansion Theorem gives as a solution of (57)

$$y = \left[\frac{Y(p)}{Z(p)}\right]_{p=0} + \left[\frac{Y(p)\epsilon^{pt}}{p(dZ/dp)}\right]_{p=p_1} + \left[\frac{Y(p)\epsilon^{pt}}{p(dZ/dp)}\right]_{p=p_2} + \cdots \quad (60)$$

where p_1, p_2, etc., are roots of $Z(p)$, that is, the values of p at which $Z(p)$ vanishes.

Before proceeding with (60), we define an arbitrary number m as follows:

$$-m^2 = pLG \quad (61)$$

or $$\sqrt{pLG} = jm. \quad (62)$$

Then (59) becomes $$Z(p) = (\sin mb)/(\sin mx). \quad (63)$$

Roots of (63) are $m = s\pi/b$ where $s = 1, 2, 3$, etc.,

and from (61) $$p_1 = -(1/LG)(\pi/b)^2$$
$$p_2 = -(1/LG)(2\pi/b)^2$$

and $$p_s = -(1/LG)(s\pi/b)^2 \text{ where } s = 1, 2, 3, \text{ etc.}$$

Now, $$dZ/dp = (dZ/dm)(dm/dp). \quad (64)$$

From (61) $$-2m\,dm = LG\,dp$$

or $$dm/dp = -LG/2m \quad (65)$$

From (63) $$\frac{dZ}{dm} = \frac{b\sin mx\cos mb - x\sin mb\cos mx}{\sin^2 mx}. \quad (66)$$

Substituting (65) and (66) in (64),

$$\frac{dZ}{dp} = \left(-\frac{LG}{2m}\right)\left[\frac{b\sin mx\cos mb - x\sin mb\cos mx}{\sin^2 mx}\right]. \quad (67)$$

From (61) and (67),

$$p\frac{dZ}{dp} = \frac{m}{2}\left[\frac{b\sin mx\cos mb - x\sin mb\cos mx}{\sin^2 mx}\right] \quad (68)$$

and $$\left[p\frac{dZ}{dp}\right]_{p=p_s} = \frac{m_s b(-1)^s}{2\sin(m_s x)} = \frac{s\pi(-1)^s}{2\sin(s\pi x/b)}. \quad (69)$$

Also $$[Y(p)/Z(p)]_{p=0} = -x/b \quad (70)$$

so that (57) becomes

$$y = -\frac{x}{b} - \sum_{s=1}^{s=\infty}\frac{2(-1)^s}{s\pi}\sin\left(\frac{s\pi x}{b}\right)\epsilon^{-(s\pi/b)^2(t/LG)}. \quad (71)$$

The third term in (56) is handled in exactly the same way and becomes

$$z = -\frac{(b-x)}{b} - \sum_{s=1}^{s=\infty}\frac{2(-1)^s}{s\pi}\sin\left[\frac{s\pi}{b}(b-x)\right]\epsilon^{-(s\pi/b)^2(t/LG)}$$

[7] Cyril N. Hoyler, "An electronic sewing machine," *Electronics*, vol. 16, pp. 90–94; 160–168; August, 1943.

$$= -1 + \frac{x}{b} + \sum_{s=1}^{s=\infty} \frac{(2)(-1)^s}{s\pi} \cos(s\pi) \sin\left(\frac{s\pi x}{b}\right) \epsilon^{-(s\pi/b)^2(t/LG)}. \quad (72)$$

Substituting (71) and (72) in (56),

$$u(x) = \frac{4E}{\pi L} \frac{1}{p} \left[\sum_{s=1}^{s=\infty} \frac{\sin\left[(2s-1)(\pi x/b)\right]}{(2s-1)} \epsilon^{-((2s-1)^2\pi^2 t)/(b^2 LG)} \right]. \quad (73)$$

But operationally

$$\frac{1}{p} = \int_{t=0}^{t=t} dt$$

so

$$u(x) = \frac{4E}{\pi L} \left(-\frac{b^2 LG}{\pi^2}\right) \sum_{s=1}^{s=\infty} \frac{\sin\left[(2s-1)(\pi x/b)\right]}{(2s-1)^3} \epsilon^{-((2s-1)^2\pi^2 t)/(b^2 LG)} \Bigg|_{t=0}^{t=t} \quad (74)$$

or

$$u(x) = \frac{4EGb^2}{\pi^3} \sum_{s=1}^{s=\infty} \frac{\sin\left[(2s-1)(\pi x/b)\right]}{(2s-1)^3} \left[1 - \epsilon^{-((2s-1)^2\pi^2 t)/(b^2 LG)}\right]. \quad (75)$$

which is (47), a solution of (46).

The Distribution of Current Along a Symmetrical Center-Driven Antenna*

RONOLD KING†, ASSOCIATE, I.R.E., AND CHARLES W. HARRISON, JR.†, ASSOCIATE, I.R.E.

Summary—**The cylindrical, center-driven antenna is analyzed as a boundary-value problem of electromagnetic theory. An integral equation in the current (originally obtained in a different way by Hallén) is derived. Its solution is outlined briefly and the general formula is given. Complete curves for the distribution of current for a wide range of lengths and ratios of length to radius are given. These include curves showing the components of current in phase with the driving potential difference and in quadrature with this, and curves giving the magnitude of the current and its phase angle referred to the driving potential difference. The conventionally assumed sinusoidal distribution of current is shown to be a fair approximation for extremely thin antennas and for thicker antennas which do not greatly exceed $\lambda/2$ in length.**

Introduction

THE distribution of current along a center-driven, symmetrical antenna of small circular cross section and half-length h is not the same as the distribution along the same conductors when these are folded together to form a closely spaced parallel-wire line of length h. Because the conductors are actually and identically the same in the two arrangements one might legitimately assume that the distributions of current would be similar. On the other hand the geometrical configuration of the two wires differs in such a fundamental way from the point of view of general electromagnetic theory that great differences in the distribution of current might also be expected. The fact is, that two parallel wires which carry equal and opposite currents sufficiently close together in terms of the wavelengths may be analyzed to a good approximation in terms of ordinary electric-circuit theory, whereas the same two wires placed end to end may not. The criterion is this: If the resultant force acting at any instant on the charges in any small element of a conductor due to charges moving at appropriate earlier times in the rest of an extended circuit includes *significant* contributions *only* from neighboring parts of the circuit (that are not more than a very small fraction of a wavelength away) then ordinary electric-circuit theory is a good approximation. In this case radiation is neglected *because it is negligible.* If the spacing b of a parallel-wire line is sufficiently small ($b \ll\ll \lambda$) then the forces on the charges in a given element ds of one of the two wires due to equal and opposite currents and charges in parallel elements which are more distant than ten times the spacing b practically cancel. All significant forces are due to charges moving in immediately adjacent parts of the two wires. In the case of the antenna no such cancellation of forces due to moving charges which are separated more than a small fraction of a wavelength occurs, and ordinary electric-circuit theory is not applicable. This is equivalent to stating that radiation is not negligible.

From the point of view of electromagnetic theory the parallel-wire line with an open end is a special case of the center-driven antenna, and it may be analyzed rigorously as such. On the other hand the antenna is in no fundamental sense a folded-open section of transmission line.

Two methods of attacking the problem of the distribution of current along the center-driven antenna suggest themselves. In the first of these one depends upon the similarity between the antenna and the open-end parallel-wire line, and assumes that by suitably correcting transmission-line theory a satisfactory approximation for the antenna may be devised. One might, for example, measure the input impedance of the

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antenna for a given value of h, and then equate this to the general formula for the input impedance of a terminated section of transmission line of length h. By suitably adjusting two or more of the parameters involved, viz., the attenuation constant α, the phase constant β, the characteristic impedance Z_c, or the terminal impedance $Z=R+jX$, an "equivalent" line can be determined. If the value of h were varied over a wide range the "equivalence" which was established at one value would be only roughly maintained but the order of magnitude would be correctly given. One might then assume that the distribution of current along the "adjusted" parallel-wire line should be a rough approximation of that along the antenna. Correction factors to be applied to line theory in order to approximate the antenna may be devised in many ways, both experimental and theoretical.[1] All such methods are, however, essentially makeshifts which may lead to results which are adequate for many engineering purposes, but which do not actually solve the problem. By skillfully devising enough correction factors any theory can always be made to fit any problem. But such methods are justified only while a more rigorous approach has not been carried out.

The second method of attacking the problem of the distribution of current along a center-driven antenna does not attempt arbitrarily to correct a theory which does not actually apply. It proceeds rather, from the point of view that the antenna is a boundary-value problem in its own right which can certainly be formulated in general terms. If it cannot be solved in closed form, it can at least be evaluated approximately in terms of parameters which characterize the antenna itself, parameters such as the length and radius of the wire, rather than in terms of a characteristic impedance, an attenuation constant, or a terminal impedance which are essentially foreign to the antenna. The antenna was investigated from this general point of view by L. V. King[2] and by Hallén[3] using different but comparable methods. Both are analytically complicated. Actually the problem can be set up formally much more directly than was done by either of these two investigators, and this will be done below. The formulation leads directly to the integral equation obtained by Hallén, rather than to that derived by King. The solution of this equation will be described only briefly because it differs in no essential way from that carried out by Hallén. Since Hallén's paper is not readily available it seems desirable to provide at least an outline of the analysis.

[1] See, for example, S. A. Schelkunoff and C. B. Feldman, "On radiation from antennas," PROC. I.R.E., vol. 30, pp. 511–516; November, 1942.

[2] L. V. King, "On the radiation field of a perfectly conducting base-insulated cylindrical antenna over a perfectly conducting plane earth, and the calculation of radiation resistance and reactance" *Phil. Trans. Royal Soc.* (London), vol. 236, pp. 381–422; November 2, 1937.

[3] E. Hallén, "Theoretical investigations into the transmitting and receiving qualities of antennas," *Nova Acta, Royal Soc. Sciences*, (Uppsala) vol. 11, pp. 1–44, November, 1938.

The Differential Equation

The analytical problem for the determination of the distribution of current in a cylindrical antenna of half-length h and radius a may be formulated in terms of the general boundary condition which requires continuity of the tangential component of the electric field across any boundary surface between two media. If the axis of the antenna is made to fall along the z axis of a system of cylindrical co-ordinates, r, θ, z, the following boundary conditions obtain:

$$(E_z^0)_{r=a} = (E_z^i)_{r=a} \text{ on the cylindrical surface} \quad (1a)$$

$$(E_r^0)_{z=\pm h} = (E_r^i)_{z=\pm h} \text{ on the end faces.} \quad (1b)$$

The superscript i refers to the interior of the conductor, the superscript 0 to space outside the conductor. The electric field in the conductor everywhere satisfies the relation

$$i = \sigma E. \quad (2)$$

Here σ is the conductivity and i is the volume density of current. In the idealized case of a perfect conductor the tangential components of E would vanish on the surface. If the end-faces are required to be small so that the following conditions are fulfilled:

$$a \ll h \quad (3a)$$

$$\beta a = (2\pi a/\lambda) \ll 1, \quad (3b)$$

then the average electric field $(E_r)_{z=\pm h}$ at the end faces must be less than the average field $(E_z)_{r=a}$ along the cylindrical surface. This follows because i_r near the end faces must vanish at $r=a$, and with (3*b*) it cannot reach a large amplitude between $r=0$ and $r=a$. Accordingly nothing of significance is neglected in so far as the antenna as a whole is concerned if no account is taken of the end faces and hence of $(E_r)_{z=\pm h}$. Thus one may assume the current to vanish at $z=\pm h$ without flowing radially inward on the end faces. (*Note added in proof:* The significance of the end faces and of the approximations involved in neglecting them has been considered by L. Brillouin in a paper which formulates the antenna problem in a mathematically more precise but very much more intricate way. For very thick cylindrical antennas this is important; for moderately thin ones as required by (3). The end faces can certainly have no greater effect than that due to an increase in h by a.)

In carrying out the analysis it will be taken for granted that the cross-sectional and axial distributions of current are mutually independent. This is always true to a very high degree of approximation in a good conductor provided (3) is fulfilled. It is commonly assumed in the derivation of the interval impedance per unit length z^i due to skin effect; it is also assumed in the derivation of the transmission-line equations. Accordingly, one may write

$$(E_z^i)_{r=a} = z^i I_z. \quad (4a)$$

Here I_z is the total current in the conductor at the cross section z, z^i is the internal impedance in ohms per meter. At high frequencies it is

$$z^i = \frac{1}{2\pi a}\sqrt{\frac{\omega\mu\Pi}{2\sigma}}. \tag{4b}$$

Here $\Pi = 4\pi \times 10^{-7}$ henry per meter; σ is the conductivity in mhos per meter; a the radius in meters, μ the relative permeability of the antenna.

The electric field at outside points is conveniently calculated from the vector potential defined by

$$\text{curl } A = B \tag{5a}$$

$$\text{div } A = -j(\omega/c^2)\phi \tag{5b}$$

using the relation defining the scalar potential ϕ. It is

$$-\text{grad } \phi = E + j\omega A. \tag{6}$$

This may be written in the form,

$$E = -\frac{jc^2}{\omega}\left(\text{grad div } A + \frac{\omega^2}{c^2}A\right) \tag{7}$$

if ϕ is eliminated from (6) using (5b).

Except at points very near the end faces one can write

$$A_r \ll A_z. \tag{8a}$$

One also has at all points,

$$A_\theta = 0. \tag{8b}$$

Accordingly the z component of the electric field has the following value except very near the end faces:

$$E_z = -j\frac{\omega}{\beta^2}\left(\frac{d^2A_z}{dz^2} + \beta^2 A_z\right). \tag{9}$$

Here

$$\beta^2 \equiv \frac{\omega^2}{c^2}. \tag{10}$$

Upon substituting (9) and (4) in (1) the following differential equation in the vector potential is obtained:

$$\frac{d^2A_z{}^0}{dz^2} + \beta^2 A_z{}^0 = j\frac{\beta^2}{\omega}z^i I_z. \tag{11}$$

The vector potential is thus seen to satisfy a one-dimensional wave equation which is homogeneous in the idealized case of an antenna which is a perfect conductor so that $z^i = 0$. It is readily verified using (5b) that the scalar potential satisfies an entirely similar equation. The total current does not satisfy such a simple equation as will be shown directly.

The Formal Solution of the Equation

The differential equation (11) is a nonhomogeneous equation which has a general solution involving the sum of a complementary function $A_c{}^0$ and a particular integral $A_p{}^0$. The former may be written in the form

$$A_c{}^0 = \frac{-j}{c}[C_1 \cos \beta z + C_2 \sin \beta z] \tag{12a}$$

with C_1 and C_2 arbitrary constants of integration. A particular integral is

$$A_r{}^0 = \frac{jz^i}{c}\int_0^z I(s) \sin \beta(z-s)ds. \tag{12b}$$

It is readily verified by substituting (12b) in (11) that it satisfies the equation. Thus the general solution of (11) is

$$A_z{}^0 = \frac{-j}{c}\left[C_1 \cos \beta z + C_2 \sin \beta z - z^i\int_0^z I(s) \sin \beta(z-s)ds\right]. \tag{13}$$

Let it be required that the antenna under consideration be symmetrical with respect to a pair of closely spaced driving points 0 and 0′ at its center in such a way that the following symmetry conditions obtain:

$$I(z) = I(-z); \quad A^0(z) = A^0(-z). \tag{14}$$

The relation (13) is easily specialized to satisfy (14) by writing $|z|$ for z in $\sin \beta z$. Thus

$$A_z{}^0 = \frac{-j}{c}\left[C_1 \cos \beta z + C_2 \sin \beta |z| - z^i\int_0^z I(s) \sin \beta(z-s)ds\right]. \tag{15}$$

It is readily verified that (15) is unchanged if $-z$ is everywhere written for z. (In the integral the variable is changed by writing $s = -u$ after writing $-z$ for z.)

The Driving-Potential Difference

Let it be assumed that a driving-potential difference $V_0{}^e$ is maintained between the two terminals 0 and 0′ which are assumed to be separated an infinitesimal distance. In practice, terminals are always separated a finite distance but it is here postulated that it is in any case a negligible fraction of a wavelength. The actual case is readily reduced to the assumed one as shown in Fig. 1. The actual terminals are A and B and a transmission line is connected to them, as shown on the left. By filling the gap between A and B in the manner shown on the right, the equal and opposite currents in the indefinitely close parallel conductors from B' to 0′ and from A' to 0 completely cancel in so far as could be determined at outside points. Thus the antenna may be assumed to extend without break across AB; it includes a point generator maintaining the potential difference $V_0{}^e$ across its terminals. In the same way the transmission line may be taken to extend from A' to B' without break with a point load concentrated midway between A' and B'.

The boundary condition on the scalar potential is

$$V_0{}^e = \lim_{z\to 0}\{\phi^0(+z) - \phi^0(-z)\}. \tag{16}$$

From (5b) one has, since $A_r \ll A_z$, $A_\theta = 0$,

$$\frac{\partial A_z{}^0}{\partial z} = -j\frac{\omega}{c^2}\phi^0. \tag{17a}$$

Also,

$$\frac{\partial A_z{}^0(+z)}{\partial z} = -j\frac{\omega}{c^2}\phi^0(+z) \tag{17b}$$

$$\frac{\partial A_z{}^0(-z)}{-\partial z} = -j\frac{\omega}{c^2}\phi^0(-z) \tag{17c}$$

so that

$$\phi^0(-z) = -\phi^0(z) = j\frac{c^2}{\omega}\frac{\partial A_z{}^0(-z)}{\partial z} \tag{18}$$

and

$$V_0{}^e = 2\lim_{z\to 0}\phi^0(+z) = \frac{2jc^2}{\omega}\lim_{z\to 0}\frac{\partial A_z{}^0(+z)}{\partial z}. \tag{19}$$

Upon differentiating (15) with respect to z and allowing z to approach zero one has

$$\lim_{z\to 0}\left(\frac{\partial A_z^0}{\partial z}\right)=\frac{-j\beta}{c}C_2 \tag{20}$$

so that with $\beta=\omega/c$, one obtains

$$C_2=\tfrac{1}{2}V_0^e. \tag{21}$$

The Integral Equation

It is shown in Appendix I that the vector potential at all points outside a cylindrical conductor (including its surface) except those within distances of an end face comparable with its radius is given to a good approximation by

$$A_z^0=\frac{\Pi}{4\pi}\int_{-h}^{+h}I_z'\frac{e^{-j\beta R}}{R}dz'. \tag{22}$$

Here R is the distance from the point (r, θ, z) outside the conductor where A_z^0 is calculated to the center of the element dz' at z' on the axis. That is,

$$R=\sqrt{(z-z')^2+r^2}. \tag{23}$$

The universal magnetic constant is

$$\Pi=4\pi\times 10^{-7}\text{ henry per meter}. \tag{24}$$

If the integral (15) is specialized to the surface of the antenna, i.e., to $r=a$, and is then substituted in (13) one obtains

$$j\frac{c\Pi}{4\pi}\int_{-h}^{+h}I_z'\frac{e^{-j\beta R}}{R}dz'=C_1\cos\beta z+\tfrac{1}{2}V_0^e\sin\beta\,|z|-z^i\int_0^z I(s)\sin\beta(z-s)ds. \tag{25}$$

In terms of the fundamental electric constant

$$\Delta=8.85\times 10^{-12}\text{ farad per meter} \tag{26}$$

and the magnetic constant defined in (24) one has

$$c=\frac{1}{\sqrt{\Pi\Delta}}=3\times 10^8\text{ meters per second} \tag{27a}$$

and,

$$R_c=\sqrt{\frac{\Pi}{\Delta}}=c\Pi=376.7\text{ ohms}. \tag{27b}$$

Then

$$c\Pi/4\pi=R_c/4\pi=30\text{ ohms}. \tag{28}$$

This may be substituted in (25). The notation in terms of R_c (ohms) will be retained so that simple dimensional relations are at all times in view.

As a first step in the solution, the integral on the left in (25) may be expanded in the following way:

$$\int_{-h}^{+h}I_z'\frac{e^{-j\beta R}}{R}dz'=I_z\int_{-h}^{+h}\frac{dz'}{R}+\int_{-h}^{+h}\frac{I_z'e^{-j\beta R}-I_z}{R}dz'. \tag{29}$$

The first integral on the right can now be evaluated directly. It is

$$\int_{-h}^{+h}\frac{dz'}{R}=\ln\left[\frac{\sqrt{(h-z)^2+a^2}+(h-z)}{\sqrt{(h+z)^2+a^2}-(h+z)}\right]. \tag{30}$$

With the notation,

$$\Omega\equiv 2\ln\left(\frac{2h}{a}\right) \tag{31}$$

$$\delta\equiv\ln\left\{\frac{1}{4}\left[\sqrt{1+\left(\frac{a}{h-z}\right)^2}+1\right]\cdot\left[\sqrt{1+\left(\frac{a}{h+z}\right)^2}+1\right]\right\}. \tag{32}$$

Equation (30) may be written as follows:

$$\int_{-h}^{+h}\frac{dz'}{R}=\Omega+\ln(1-z^2/h^2)+\delta. \tag{33}$$

It is to be noted that δ is negligible except very near the ends of the antenna and that (30), (which includes δ) is everywhere finite reducing to the following value at the ends:

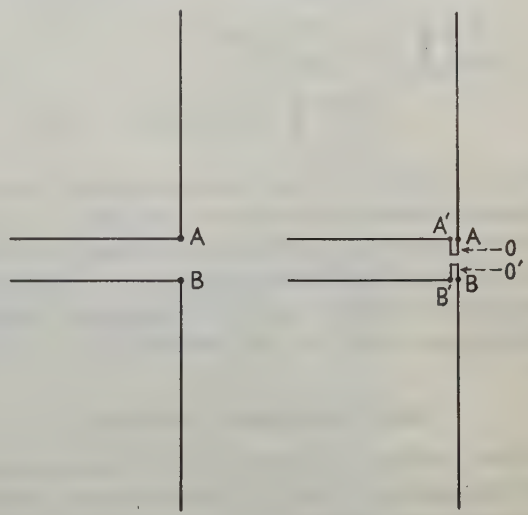

Fig. 1—Actual method of feeding and analytically equivalent method.

$$\left[\int_{-h}^{+h}\frac{dz'}{R}\right]_{z=\pm h}=\tfrac{1}{2}\Omega+\ln 2. \tag{34}$$

Upon substituting (33) in (29) and then inserting (29) in (25) one readily obtains

$$I_z=\frac{-j4\pi}{\Omega R_c}\left\{C_1\cos\beta z+\tfrac{1}{2}V_0^e\sin\beta\,|z|-z^i\int_0^z I(s)\sin\beta(z-s)ds\right\}-\frac{1}{\Omega}\left\{I_z\ln(1-z^2/h^2)+I_z\delta+\int_{-h}^{+h}\left(\frac{I_z'e^{-j\beta R}-I_z}{R}\right)dz'\right\}. \tag{35}$$

Since the current vanishes at the ends and $\ln(1-z^2/h^2)+\delta$ remains finite according to (33) and (34), one has, with $z=h$,

$$0=\frac{-j4\pi}{\Omega R_c}\left\{C_1\cos\beta h+\tfrac{1}{2}V_0^e\sin\beta h\right\}-\frac{1}{\Omega}\left\{\frac{j4\pi z^i}{R_c}\int_0^h I(s)\sin\beta(h-s)ds-\int_{-h}^{+h}I_z'\frac{e^{-j\beta R_h}}{R_h}dz'\right\}. \tag{36}$$

Here

$$R_h=\sqrt{(h-z')^2+a^2}. \tag{37}$$

If (36) is subtracted from (35) one has

$$I_z = \frac{-j4\pi}{\Omega R_c}\{C_1(\cos\beta z - \cos\beta h) + \tfrac{1}{2}V_0^e(\sin\beta|z| - \sin\beta h)\} - \frac{1}{\Omega}\left\{I_z \ln(1 - z^2/h^2) + I_z\delta + \int_{-h}^{+h}\left(\frac{I_z' e^{-j\beta R} - I_z}{R}\right)dz' - \frac{j4\pi z^i}{R_c}\int_0^z I(s)\sin\beta(z-s)ds\right\} + \frac{1}{\Omega}\left\{\int_{-h}^{+h} I_z'\frac{e^{-j\beta R_h}}{R_h}dz' - \frac{j4\pi z^i}{R_c}\int_0^h I(s)\sin\beta(h-s)ds\right\}. \quad (38)$$

This expression is the same as that originally derived by Hallén in a somewhat different way. Since its evaluation from this point on follows in all essential respects the method used by Hallén it will not be reproduced in detail. A brief outline is given in Appendix II because Hallén's paper is not generally available at the present time.

The First-Order Solution

By the method of successive approximations outlined in Appendix II, (38) may be expressed in the form of a series in the small quantity $1/\Omega$. By substituting this series in (36) the constant of integration C_1 may be evaluated. The zeroth and first-order terms in the solution are

$$I_z = \frac{j2\pi V_0^e}{\Omega R_c}\left\{\frac{\sin\beta(h-|z|)+(1/\Omega)\,[M_1^I + jM_1^{II}]}{\cos\beta h + (1/\Omega)\,[A_1^I + jA_1^{II}]}\right\}. \quad (39)$$

Terms involving factors of order $1/\Omega^2$, $1/\Omega^3$, etc., are neglected in (39). The real functions M_1^I, M_1^{II}, A_1^I, and A_1^{II}, which are functions of h and z only and not at all of the radius a, are defined as follows in terms of the complex F and G functions given in Appendix II.

$$M_1^I + jM_1^{II} \equiv F_1(z)\sin\beta h - F_1(h)\sin\beta|z| + G_1(h)\cos\beta z - G_1(z)\cos\beta h \quad (40)$$

$$A_1^I + jA_1^{II} \equiv F_1(h). \quad (41)$$

These have been computed for several values of h as shown graphically in Figs. 2 to 6.

Let the numerator in the brace of (39) be denoted by

$$N^I + jN^{II} = Ne^{j\psi_N} \quad (42a)$$

with

$$N^I = \sin\beta(h - |z|) + M_1^I/\Omega; \quad N^{II} = M_1^{II}/\Omega. \quad (42b)$$

Similarly let the denominator in (39) be

$$D^I + jD^{II} = De^{j\psi_D} \quad (42c)$$

with $$D^I = \cos\beta h + A_1^I/\Omega; \quad D^{II} = A_1^{II}/\Omega. \quad (42d)$$

Also let $$f' = N\cos(\psi_D - \psi_N) \quad (42e)$$

$$f'' = N\sin(\psi_D - \psi_N). \quad (42f)$$

Both f' and f'' are functions of z, while D is not. With this notation, (39) reduces to

$$I_z = \frac{2\pi V_0^e}{\Omega R_c D}(f'' + jf') = \frac{V_0^e}{60\Omega D}(f'' + jf'). \quad (43)$$

In amplitude-phase-angle form one has

$$I_z = \frac{V_0^e}{60\Omega D}\sqrt{(f'')^2 + (f')^2}\,e^{j\theta} \quad (44)$$

with $$\theta = \tan^{-1}\left(\frac{f'}{f''}\right). \quad (45)$$

If the applied voltage varies according to

$$v_0^e = V_0^e \sin\omega t \quad (46)$$

then the instantaneous current at a distance z from the center of the symmetrical center-driven antenna is

$$i_z = \frac{V_0^e}{60\Omega D}\sqrt{(f'')^2 + (f')^2}\sin(\omega t + \theta). \quad (47)$$

The distribution of current along a cylindrical antenna which satisfies the condition

$$\Omega^2 \gg 1 \quad (48)$$

has thus been obtained.

If (48) is interpreted to mean

$$\Omega \geqq 10, \quad (49)$$

one can also write as an equivalent

$$\frac{h}{a} \geqq 75. \quad (50)$$

Curves showing the functions f'' and f' for use with the formula (43) are reproduced in Figs. 7 to 12 for several lengths and three different thicknesses covering most of the practical range. Actually it is merely necessary to multiply the values of f'' or f' in the curves by $1/60\Omega D$ in order to obtain the corresponding components of current in amperes per input volt. Numerical values of this factor for the several cases plotted in the figures are given in Table I. Curves giving $\sqrt{(f'')^2+(f')^2}$ and $\theta = \tan^{-1}(f'/f'')$ for use with (44) are shown in Figs. 13 to 18. Thus Figs. 7 to 18 together with (43) and (44) completely characterize the distribution of current along a typical center-fed antenna of circular cross section with radius a and of length h. Before discussing these general results it is well to consider first the input impedance and then two special cases.

The Input Impedance

In considering the significance of the distribution curves for current it is instructive to examine simultaneously the input impedance of the antenna. This is defined simply as the potential difference V_0^e maintained at the input terminals divided by the input current. It may be obtained directly from (39) by writing $z = 0$. Thus

$$Z_{00} = \frac{V_0^e}{I_0} = \frac{-j\Omega R_c}{2\pi}\left\{\frac{\cos\beta h + (1/\Omega)\,(A_1^I + jA_1^{II})}{\sin\beta h + (1/\Omega)\,(B_1^I + jB_1^{II})}\right\} \quad (51)$$

with

$$B_1^I + iB_1^{II} = F_1(0)\sin\beta h + G_1(h) - G_1(0)\cos\beta h. \quad (52)$$

TABLE I

$\frac{h}{\lambda}$	H (radians)	$\frac{1}{60\Omega D}$ (multiply all numbers by 10^{-3})			$Y_{00}=G_{00}-jB_{00}$ (in mhos $\times 10^{-3}$)		
		$\Omega=10$	$\Omega=20$	$\Omega=30$	$\Omega=10$	$\Omega=20$	$\Omega=30$
0.2425	1.538	13.396	14.094	13.805	15.54 $-j3.67$	15.36 $-j0.0$	13.8 $+j3.67$
0.25	1.571	11.819	11.815	11.810	12.74 $-j5.9$	11.39 $-j5.90$	11.04 $-j5.9$
0.375	2.356	2.320	1.176	0.785	0.992 $-j1.443$	0.255 $-j0.805$	0.115 $-j0.552$
0.50	3.141	1.820	0.870	0.589	0.574 $+j0.423$	0.140 $+j0.095$	0.066 $+j0.041$
0.625	3.927	2.806	1.284	0.830	0.834 $+j2.806$	0.174 $+j1.11$	0.073 $+j0.675$
0.75	4.712	8.670			10.2 $-j3.54$		

This is exactly the expression from which curves for the input impedance have been computed.[4]

The Distribution of Current for an Indefinitely Thin Antenna

The distribution of current along an indefinitely thin antenna is obtained from (39) by allowing the radius a to approach zero. This is equivalent to allowing the parameter Ω to approach infinity. Let the applied potential difference V_0^e be increased with Ω so that the ratio V_0^e/Ω remains finite. One then has

$$I_z = \frac{j2\pi V_0^e}{\Omega R_c} \frac{\sin \beta(h-|z|)}{\cos \beta h}. \tag{53}$$

The input impedance is formally expressed by

$$Z_{00} = jX_{00} = -j\Omega\,(R_c/2\pi)\cot \beta h. \tag{54}$$

(It is to be noted that (53) and (54) are actually good approximations for an antenna of very small but nonvanishing radius over those limited parts of the ranges of βh and $\beta(h-|z|)$ for which the trigonometric factors in both numerator and denominator in (39) and in (51) are large compared with the magnitudes of the factors involving $1/\Omega$.)

The distribution of current along an indefinitely thin antenna is seen to be very simple in form. Referred to the input current I_0, defined by

$$I_0 = j\frac{2\pi V_0^e}{\Omega R_c}\tan \beta h, \tag{55}$$

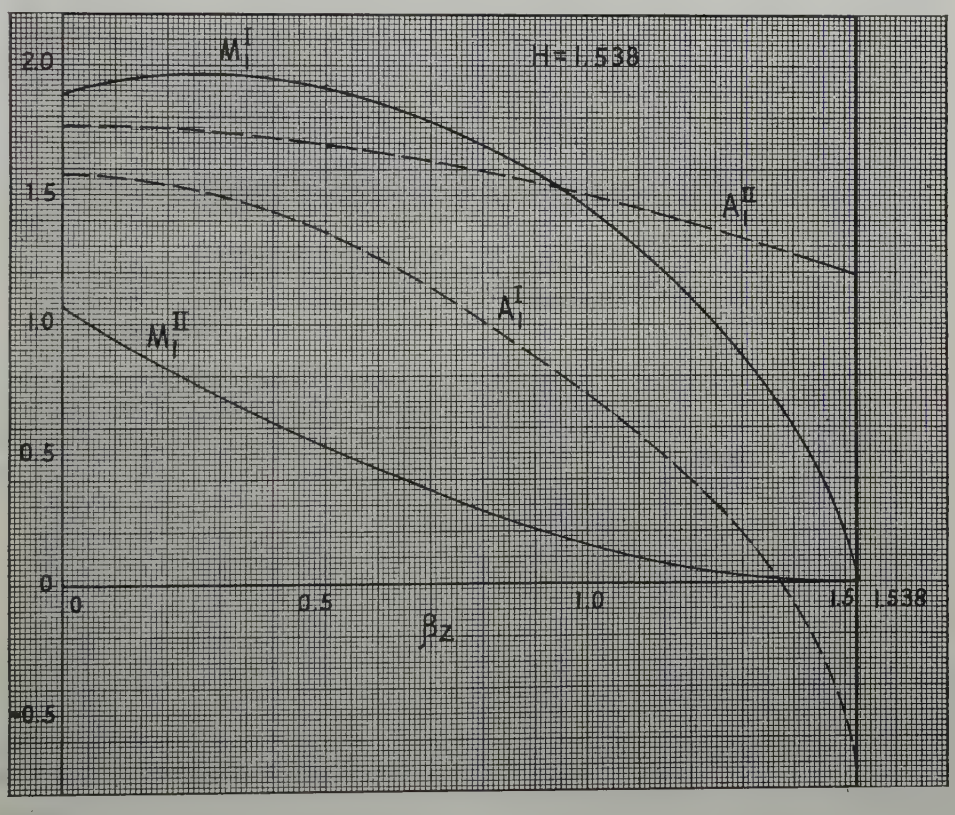

Fig. 2—The function A_1^I, A_1^{II}, M_1^I, and M_1^{II} for $H=1.538$.

it is

$$I_z = I_0\frac{\sin \beta(h-|z|)}{\sin \beta h}. \tag{56}$$

Or in terms of the maximum value defined by

$$I_{\max} = I_0/\sin \beta h \tag{57}$$

it is

$$I_z = I_{\max}\sin \beta(h-|z|). \tag{58}$$

The current I_{max} is fictitious in all antennas for which h is shorter than $\lambda/4$.

The distribution (56), or its equivalent (58), is the one usually assumed for all straight antennas regardless of radius. It is here shown to be strictly correct for an antenna of indefinitely small radius. Distribution

[4] Ronold King and F. G. Blake, "The self-impedance of a symmetrical antenna," Proc. I.R.E., vol. 30, pp. 335–349; July, 1942.

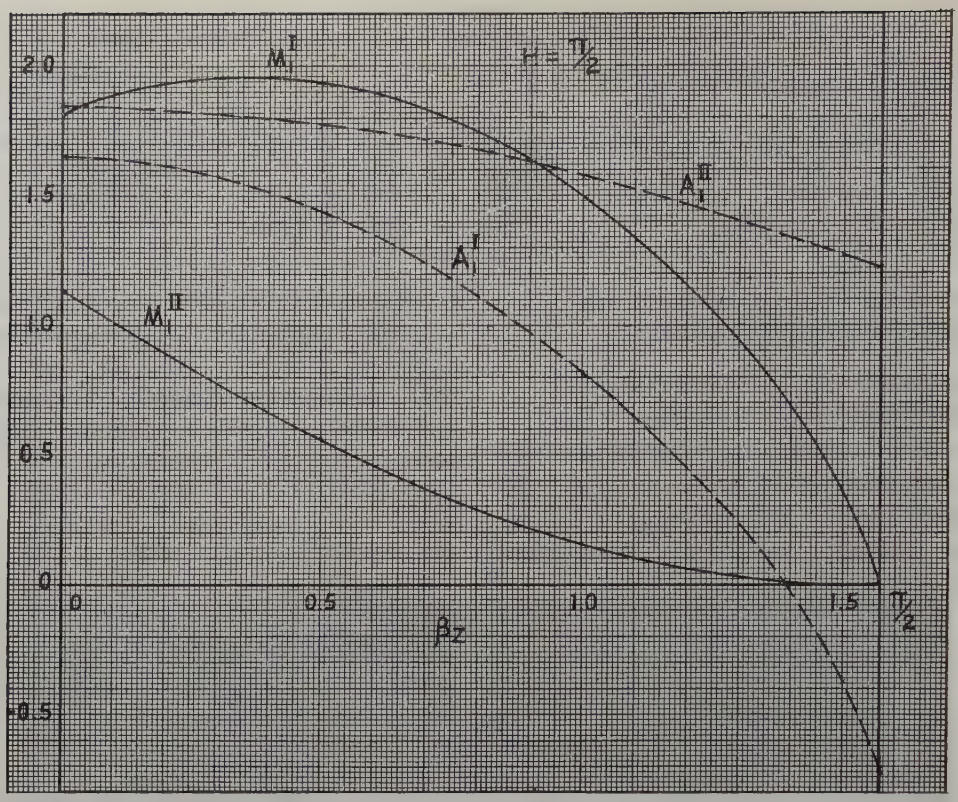

Fig. 3—The functions A_1^I, A_1^{II}, M_1^I, and M_1^{II} for $H=\pi/2$.

curves computed from (58) are well known. A few are shown in Figs. 13 to 18 marked sine curve.

The input impedance of an infinitely thin antenna as given by (54) with Ω increasing without limit requires X_{00} to be negatively infinite for βh between 0 and $\pi/2$, π and $3\pi/2$, etc., and positively infinite for βh between $\pi/2$ and π, $3\pi/2$ and 2π, etc. The values at $\beta h=\pi/2$, π, $3\pi/2$, etc., are indeterminate. The formula (54) would be a good approximation for an extremely thin antenna except near the values of βh listed above. It is not a good approximation for thick antennas.

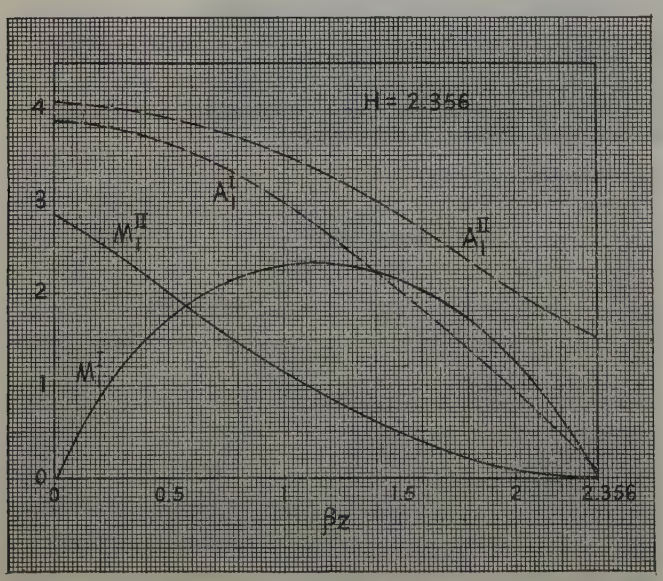

Fig. 4—The functions A_1^I, A_1^{II}, M_1^I, and M_1^{II} for $H=3\pi/4$.

The Distribution of Current for an Antenna Approximately a Half Wavelength Long

The simple sinusoidal form (53) for the distribution of current and the equally simple expression (54) for the input impedance are not useful at $\beta h=n\pi/2$ with n any integer even for indefinitely thin antennas. At $\beta h=n\pi/2$, one has from (39) and (51)

$$I_z = \frac{j2\pi V_0^e}{R_e}\left\{\frac{\cos\beta z + (1/\Omega)\,[M_1^I + jM_1^{II}]}{A_1^I + jA_1^{II}}\right\} \tag{59}$$

$$Z_{00} = \frac{-jR_e}{2\pi}\left\{\frac{A_1^I + jA_1^{II}}{1+(1/\Omega)\,(B_1^I + jB_1^{II})}\right\}. \tag{60}$$

These formulas are limited only by (48).

Since the functions A, B, and M appearing in (59) and (60) depend upon the radius a only through terms involving the ohmic resistance (which are negligible in good conductors), it follows that (59) and (60) depend upon the radius only through the one term in which $1/\Omega$ appears as a factor. That means that the distribution of current (59) and the input impedance (60) of antennas for which $h=\lambda/4$ will vary only slightly with radius as compared with antennas of other lengths which depend on the general expressions (39) and (51) that have Ω as a factor in all terms. Thus, one might expect that a reasonably satisfactory approximation for a moderately thin antenna would be given by neglecting the terms in $1/\Omega$ in (59) and (60). If this is done one obtains formulas which are *independent* of the radius, and which are strictly accurate in the limit as the radius is made increasingly small

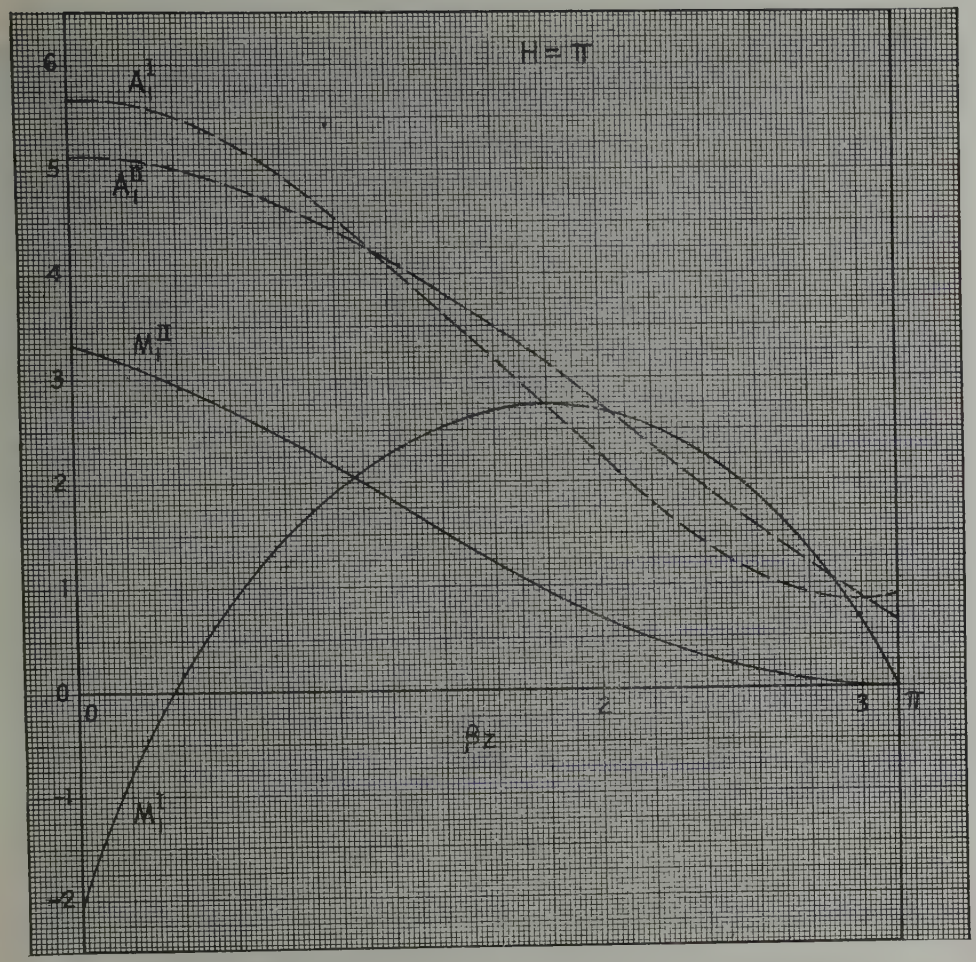

Fig. 5—The functions $A_1{}^I$, $A_1{}^{II}$, $M_1{}^I$, and $M_1{}^{II}$ for $H=\pi$.

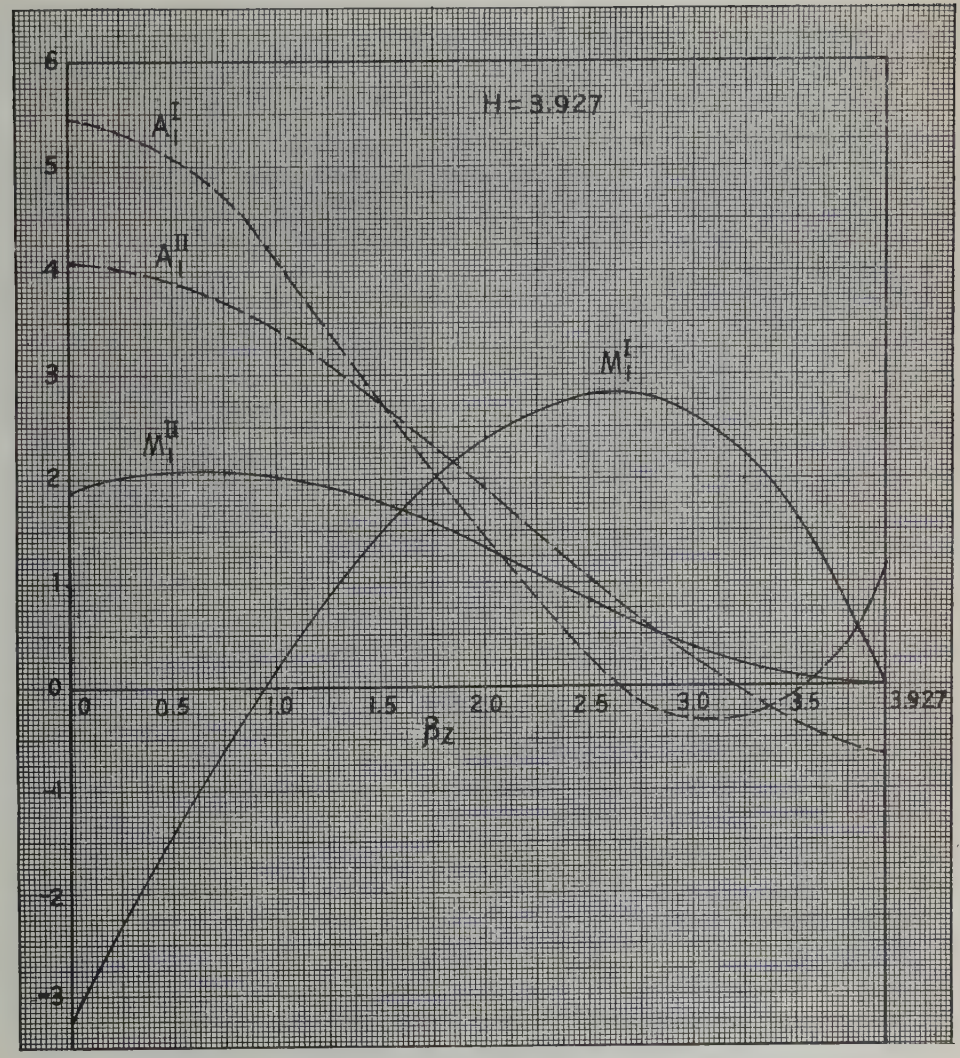

Fig. 6—The functions $A_1{}^I$, $A_1{}^{II}$, $M_1{}^I$, and $M_1{}^{II}$ for $H=5\pi/4$.

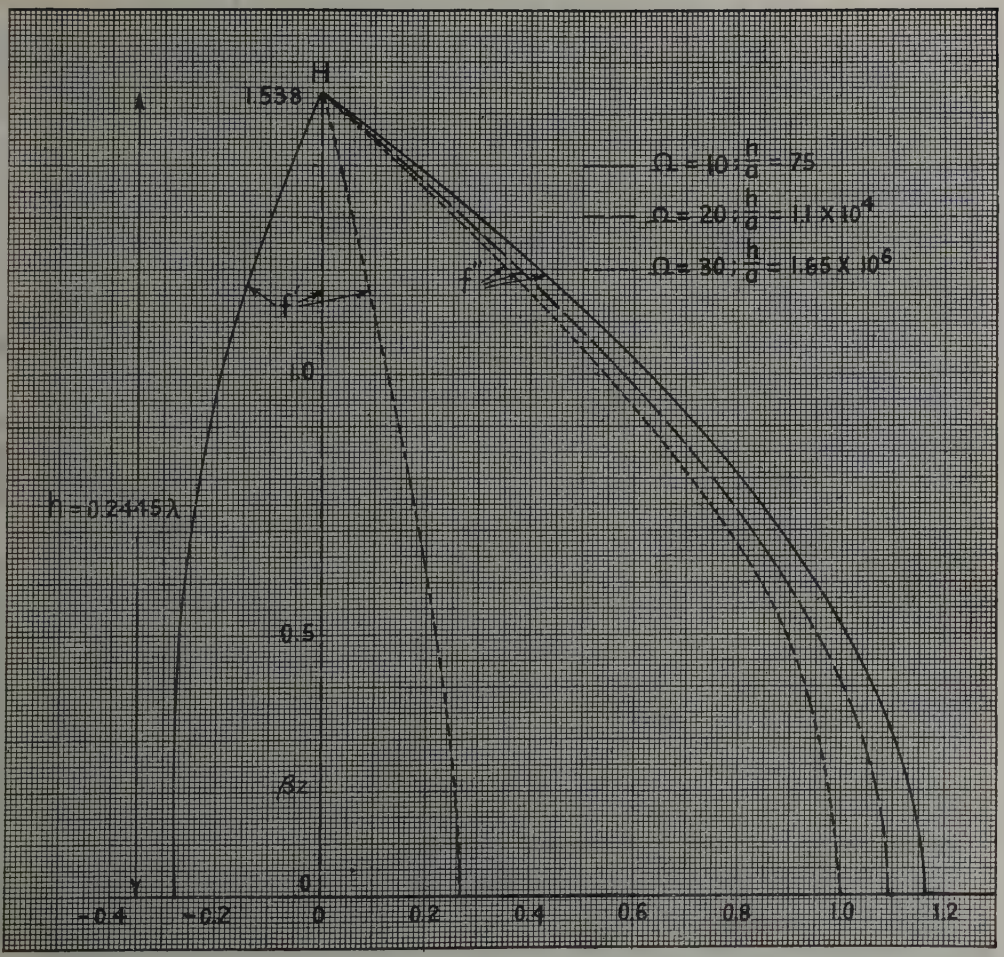

Fig. 7—The distribution functions f' and f'' for $H=1.538$.

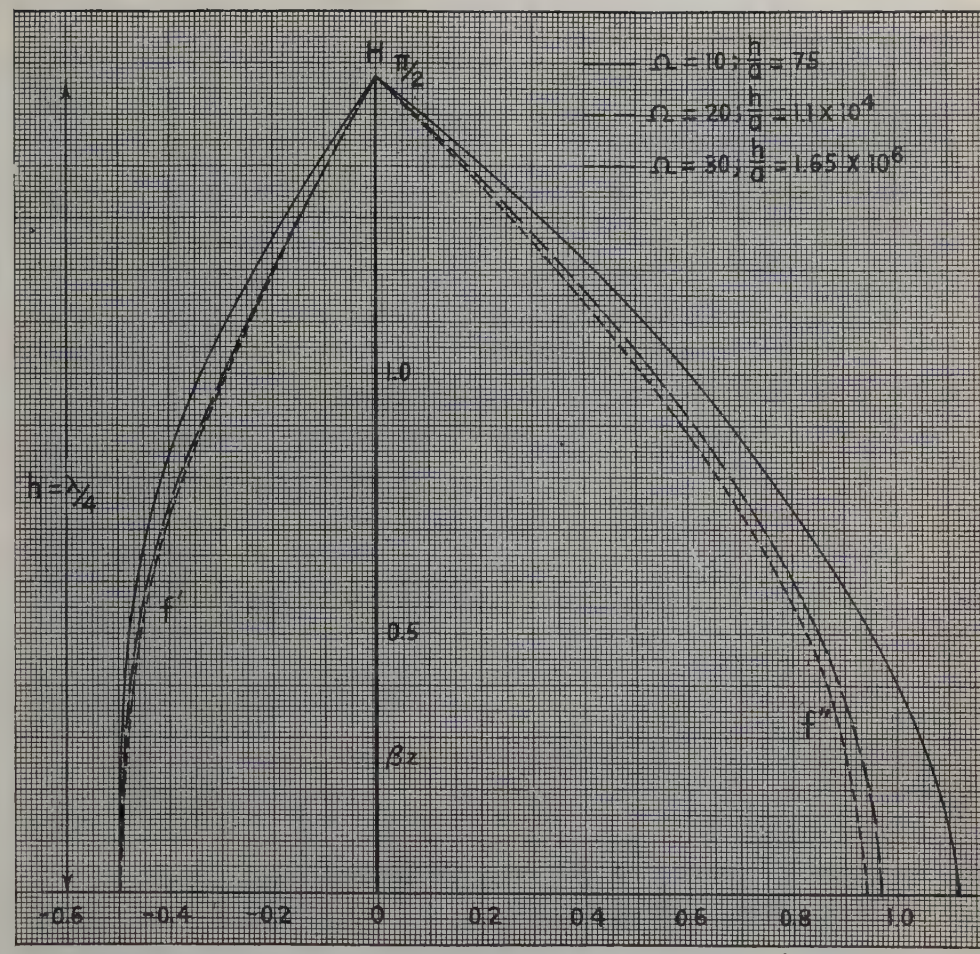

Fig. 8—The distribution functions f' and f'' for $H=\pi/2$.

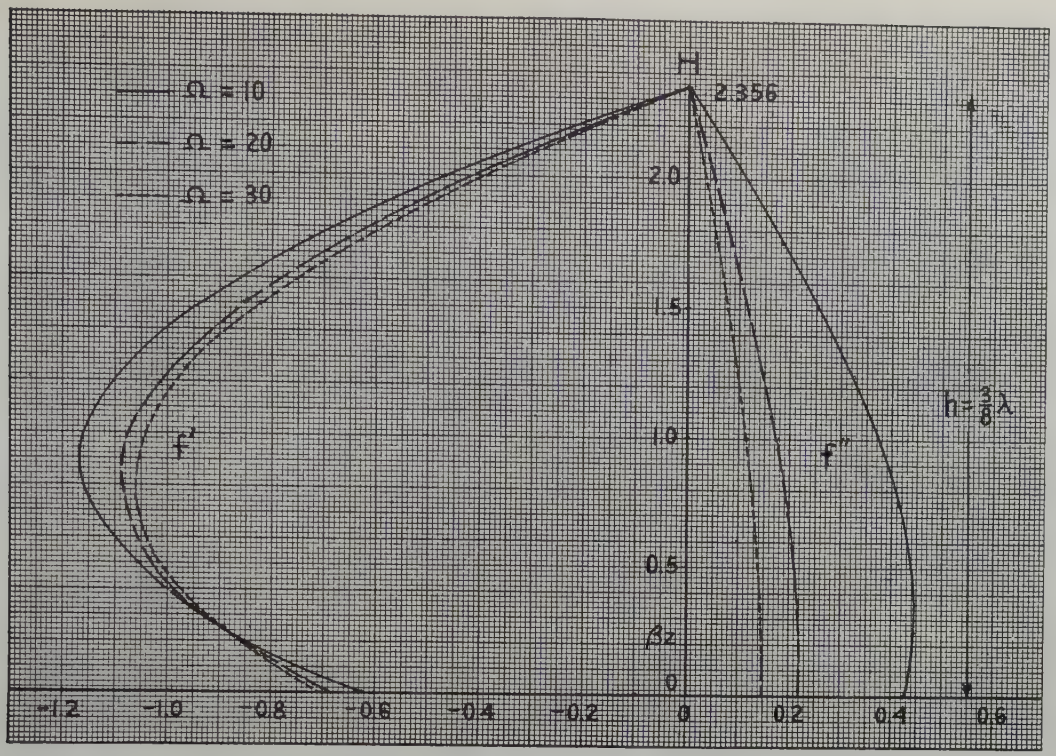

Fig. 9—The distribution functions f' and f'' for $H=3\pi/4$.

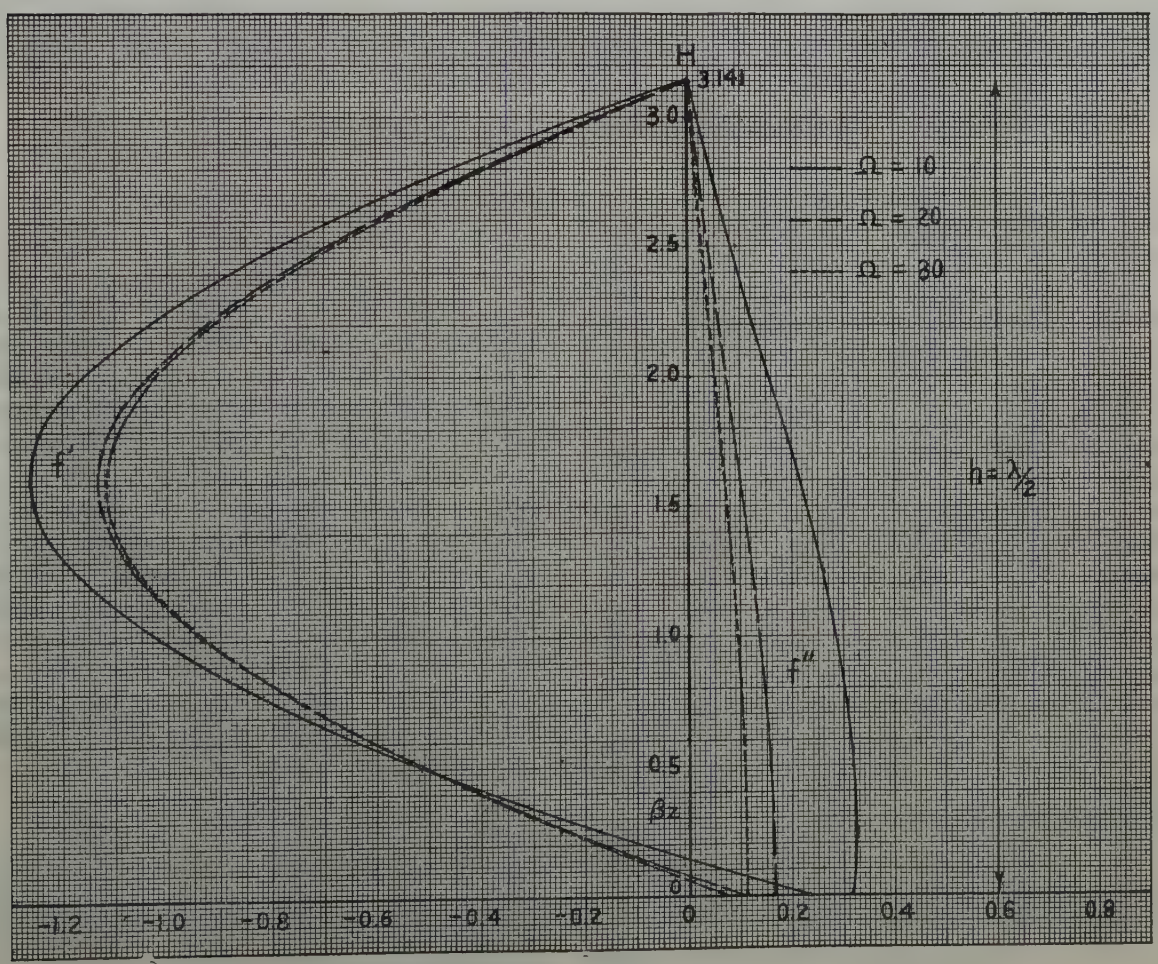

Fig. 10—The distribution functions f' and f'' for $H=\pi$.

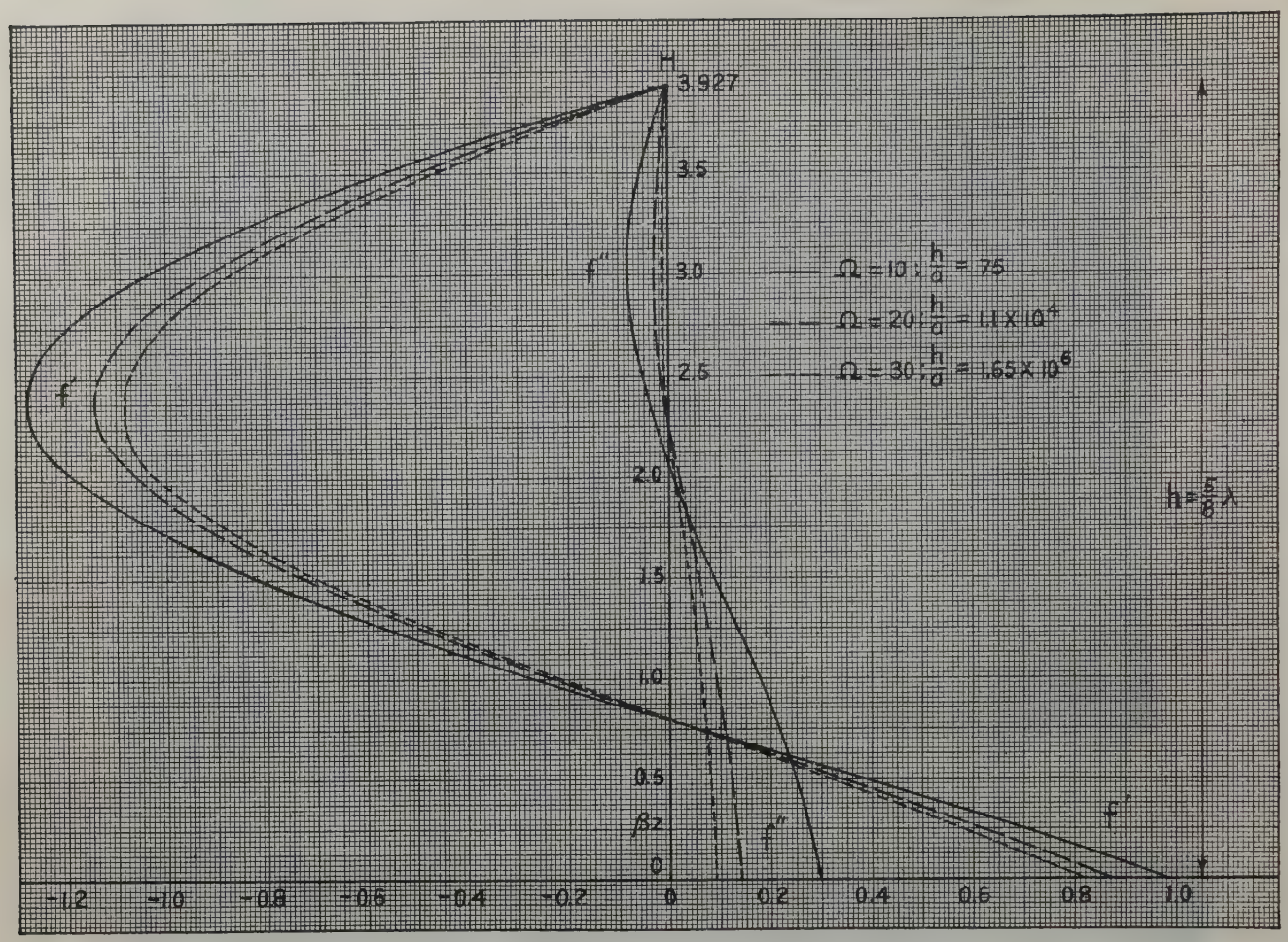

Fig. 11—The distribution functions f' and f'' for $H=5\pi/4$.

after βh has been fixed at $n\pi/2$. It can be seen from the left half of Fig. 8 in footnote reference 4 that as the ratio a/λ becomes smaller and smaller the input-reactance curve approaches the vertical and its intersection with the $X_{00}=0$ axis moves to the right. Just before one passes to the actual limit, $a=0$, the reactance curve intersects the $X_{00}=0$ axis an infinitesimal distance to the left of $\beta h=\pi/2$, while its value at $\beta h=\pi/2$ is just a trifle below $X_{00}=42.5$. In the limit, the reactance curve is a vertical line from $-\infty$ to $+\infty$ at $\beta h=\pi/2$, so that values of $X_{00}=0$ or $X_{00}=42.5$ are equally correct but actually meaningless. The formulas which apply to $\beta h=\pi/2$ with the radius a approaching, but not quite reaching, zero are given below. They are not correct for the condition of resonance $X_{00}=0$, which occurs indefinitely near $\beta h=\pi/2$, as $a\rightarrow 0$ but considerably below this value even for small radii.

$$I_z = (V_0^e/Z_{00}) \cos \beta z \tag{61}$$

with $$Z_{00} = R_{00} + jX_{00} = (R_c/2\pi)\,(A_1^{II} - jA_1^{I}). \tag{62}$$

For $\beta h=\pi/2$ the numerical value is

$$Z_{00} = 73.13 + j42.5 = 84.5 \angle 30°.2. \tag{63}$$

The formula (61) is as simple in form as (53) and permits writing

$$I_z = I_0 \cos \beta z \tag{64}$$

with I_0 now complex and given by

$$I_0 = V_0^e/Z_{00} \tag{65}$$

instead of the pure imaginary defined by (55). Thus (56) and (58) apply to an almost infinitely thin antenna with $\beta h=\pi/2$ and with $I_0=I_{max}$ defined above.

The simple formula (61) is not strictly applicable to antennas of practical thickness any more than is (53). However, because $1/\Omega$ appears in the more correct formula (59) only in one small term, (61) is a better approximation for the case $\beta h=\pi/2$ than is (53) for other values of βh. Indeed, an examination of Figs. 8 and 14, (which give the distribution of current along antennas for which $\beta h=\pi/2$ and Ω has the values 10, 20, and 30) reveals that even for the thickest antenna ($\Omega=10$), I_z does not differ greatly from a simple cosine as given by (61), but with Z_{00} standing for the actual input impedance calculated from (60). These are

$$\Omega = 10, \quad Z_{00} = 64.8 + j29.7 = 71.2 \angle 24°.6 \tag{66a}$$

$$\Omega = 20, \quad Z_{00} = 69.6 + j35.7 = 78.6 \angle 27°.2 \tag{66b}$$

$$\Omega = 30, \quad Z_{00} = 70.3 + j37.6 = 79.8 \angle 28°.2 \tag{66c}$$

Although the phase angle θ of the current as shown in Fig. 14 is not perfectly constant over the length of the antenna it varies only a few degrees from the value at the input terminals.

Antennas in General

If an antenna differs even slightly in half length from a quarter wavelength for which $\beta h=\pi/2$ it is not at all clear from (39) that an approximate formula of the type (61) or (53) may be used unless the antenna is infinitely thin. Because the term in $1/\Omega$ in the denominator of (39) is small, the term $\cos \beta h$ will be significant even though βh differs very little from $\pi/2$. The impedance formula (51) also changes very rapidly in the vicinity of $\beta h=\pi/2$. Nevertheless the actual computation of I_z for βh sufficiently below $\pi/2$ so that an antenna for which $\Omega=20$ is self-resonant with $X_{00}=0$ (Figs. 7 and 13) shows that $|I_z|$ differs but little from a sine curve measured from the upper ends with $h-|z|$ as variable, and that the phase angle θ stays very nearly constant at a value near that for I_0 even for thick antennas. On the other hand, antennas which are sufficiently long so that βh appreciably exceeds $\pi/2$, as in Figs. 15 to 18, $|I_z|$ cannot be represented very satisfactorily by a simple sine curve nor does the phase angle θ remain constant at anywhere near its value for I_0. One must conclude, therefore, that the distribution of current along antennas only of such lengths that βh does not exceed appreciably the value $\pi/2$ may be represented with fair accuracy by

$$I_z = I_0 \frac{\sin \beta(h-|z|)}{\sin \beta h} \tag{67}$$

with

$$I_0 = V_0^e/Z_{00}. \tag{68}$$

Here Z_{00} is the input impedance computed from the accurate formula (51) or obtained from the curves given in footnote reference 4. If the half-length of the antenna is much greater than $\lambda/4$, in particular, if it approaches or exceeds $\lambda/2$, a representation in terms of (67) and (68) is not satisfactory.

Since a single distribution function with a constant phase angle is not in general adequate, one is faced with the necessity of complicating the representation. Clearly a much better approximation at the expense of only a small increase in complexity would result if each component I_z' and I_z'' in

$$I_z = I_z'' + jI_z' = \frac{V_0^e}{60\Omega D}(f'' + jf') \tag{69}$$

were separately represented by a simple trigonometric function. This is actually possible to a very satisfactory degree of approximation. The representation is the following:

$$I_z = V_0^e\left[G_{00}\left(\frac{\cos \beta z - \cos H}{1-\cos H}\right) - jB_{00}\left(\frac{\sin(H-\beta|z|)}{\sin H}\right)\right]. \tag{70}$$

Here $H=\beta h$, G_{00} is the input conductance, and B_{00} the input susceptance of the antenna in question. The admittance, $1/Z_{00}=Y_{00}=G_{00}-jB_{00}$, for each of the several lengths considered above is given in Table I. In order to show that (70) is a good representation of the actual distribution one notes in the first place that the input current I_0 is exactly right for all lengths. The distribution along the antenna given by (70) may be compared with the actual distributions using Figs. 19 and 20. These show the true distribution functions, f' and f'', as obtained from Figs. 8 to 12 each plotted with the appropriate trigonometric function which is supposed to represent it approximately in (70). (In Figs. 19 and 20, f' and f'' have been adjusted in scale so that the values at $z=0$ coincide. Actually the admittance factors G_{00} and $-B_{00}$ in (70) serve to change the scales of the trigonometric functions respectively to give the correct values at $z=0$. For purposes of plotting and comparison it was more convenient to adjust f' and f'' to the trigonometric functions rather than vice versa.) It is seen that the representation of the analytically extremely complex functions f' and f'' in terms of the simple trigonometric functions is surprisingly good over practically the entire range of lengths shown. The poorest approximation is near $h=\frac{3}{8}$ for $\Omega=10$. Some difficulty in representing f' is encountered at $h=\lambda/2$, as shown in Fig. 19. Because no antenna of physically realizable radius is antiresonant at $h=\lambda/2$, B_{00} does not vanish and the imaginary term in (70) becomes infinite. This difficulty can be avoided by introducing a fictitious

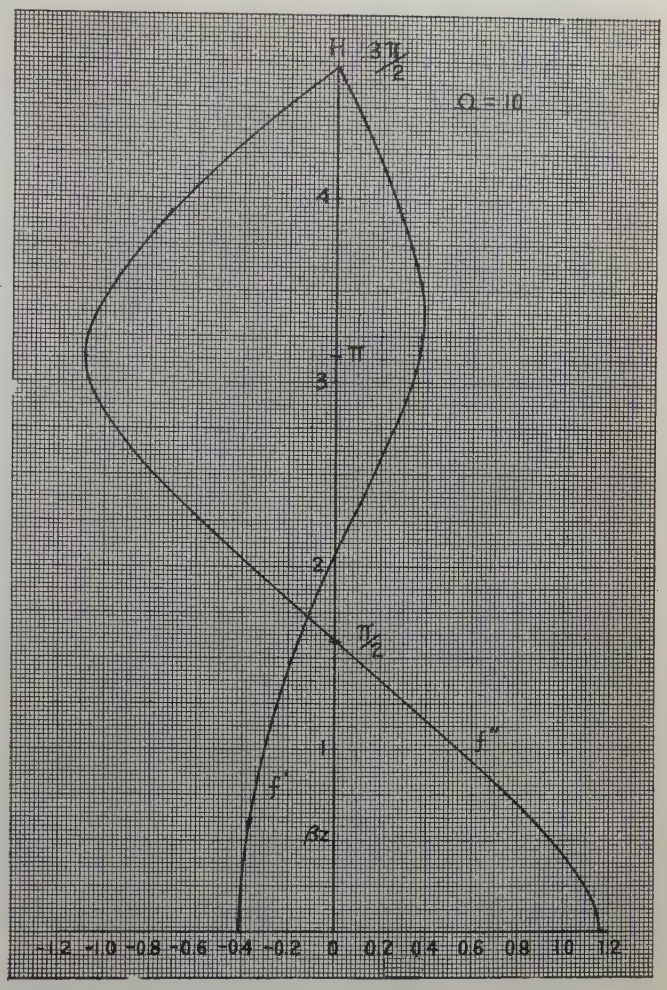

Fig. 12—The distribution functions f' and f'' for $H=3\pi/2$.

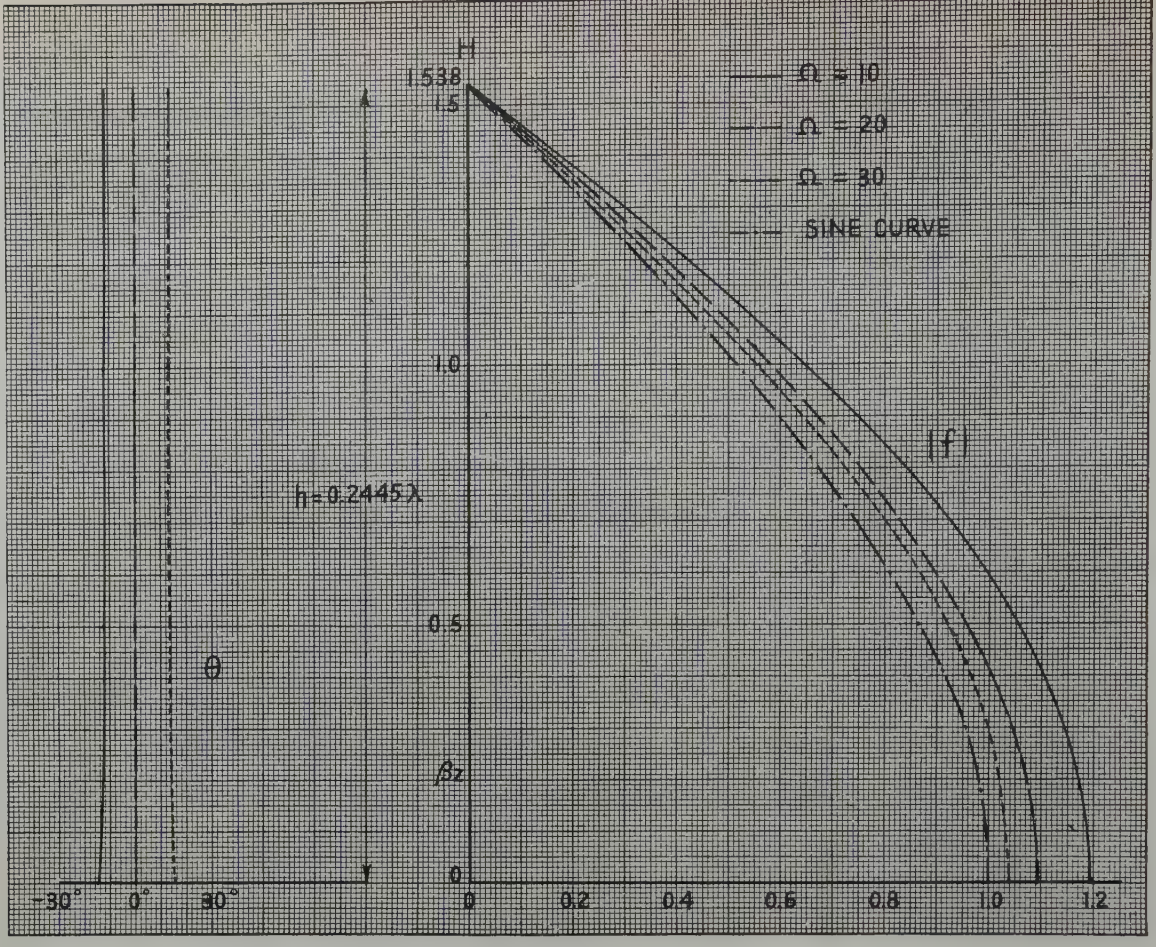

Fig. 13—The amplitude function $|f|$ and the phase function θ for $H=1.538$.

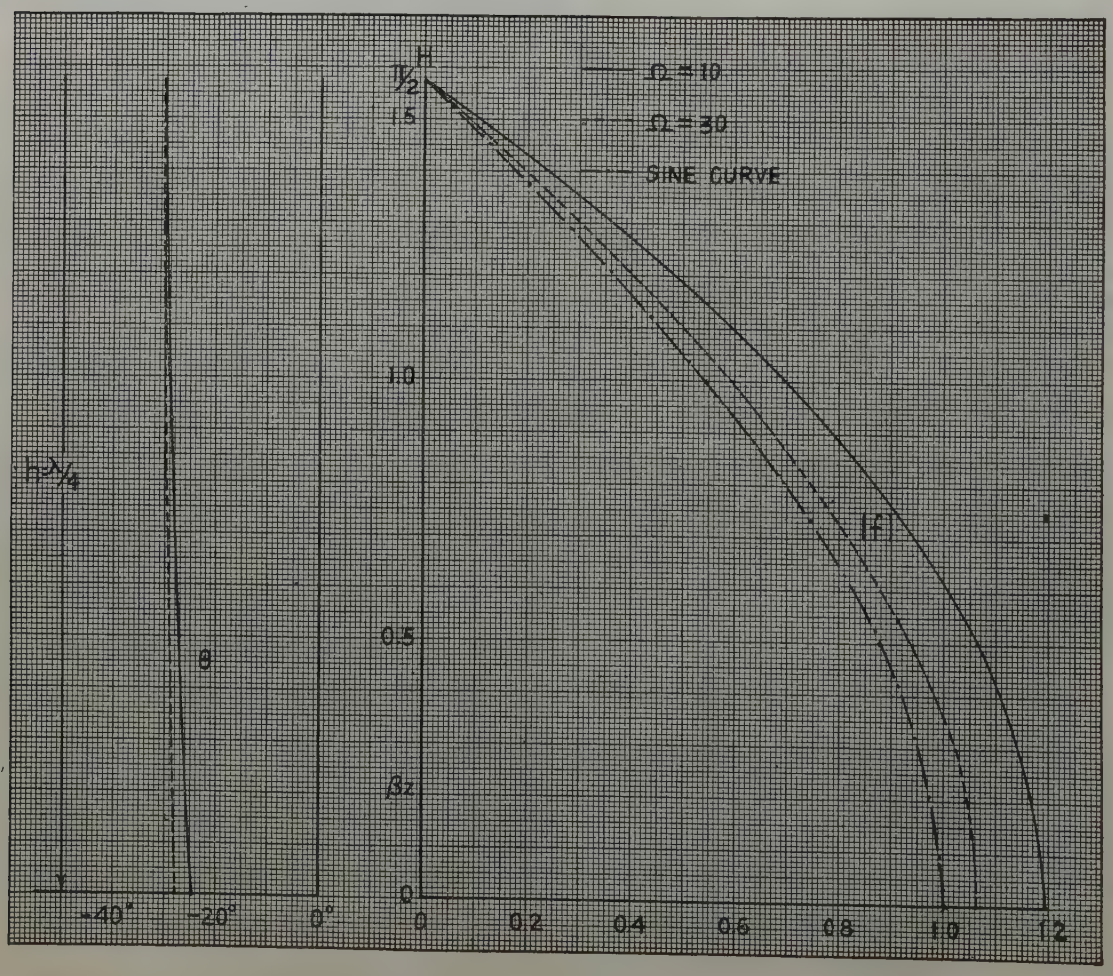

Fig. 14—The amplitude function $|f|$ and the phase function θ for $H=\pi/2$.

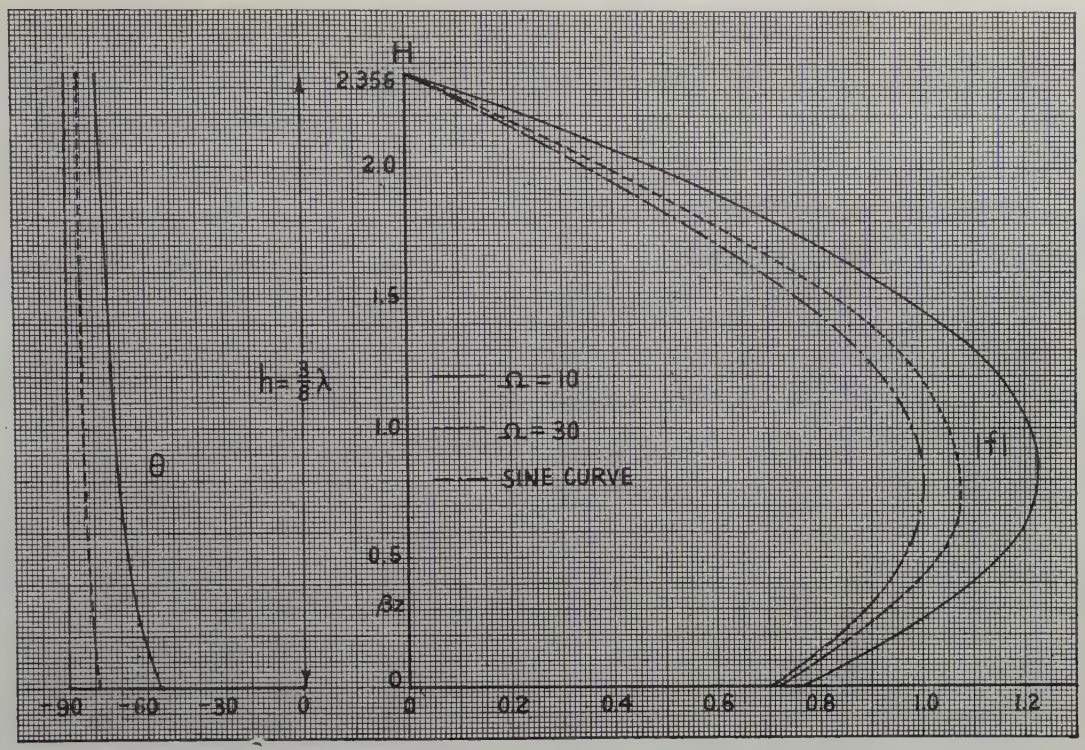

Fig. 15—The amplitude function $|f|$ and the phase function θ for $H=3\pi/4$.

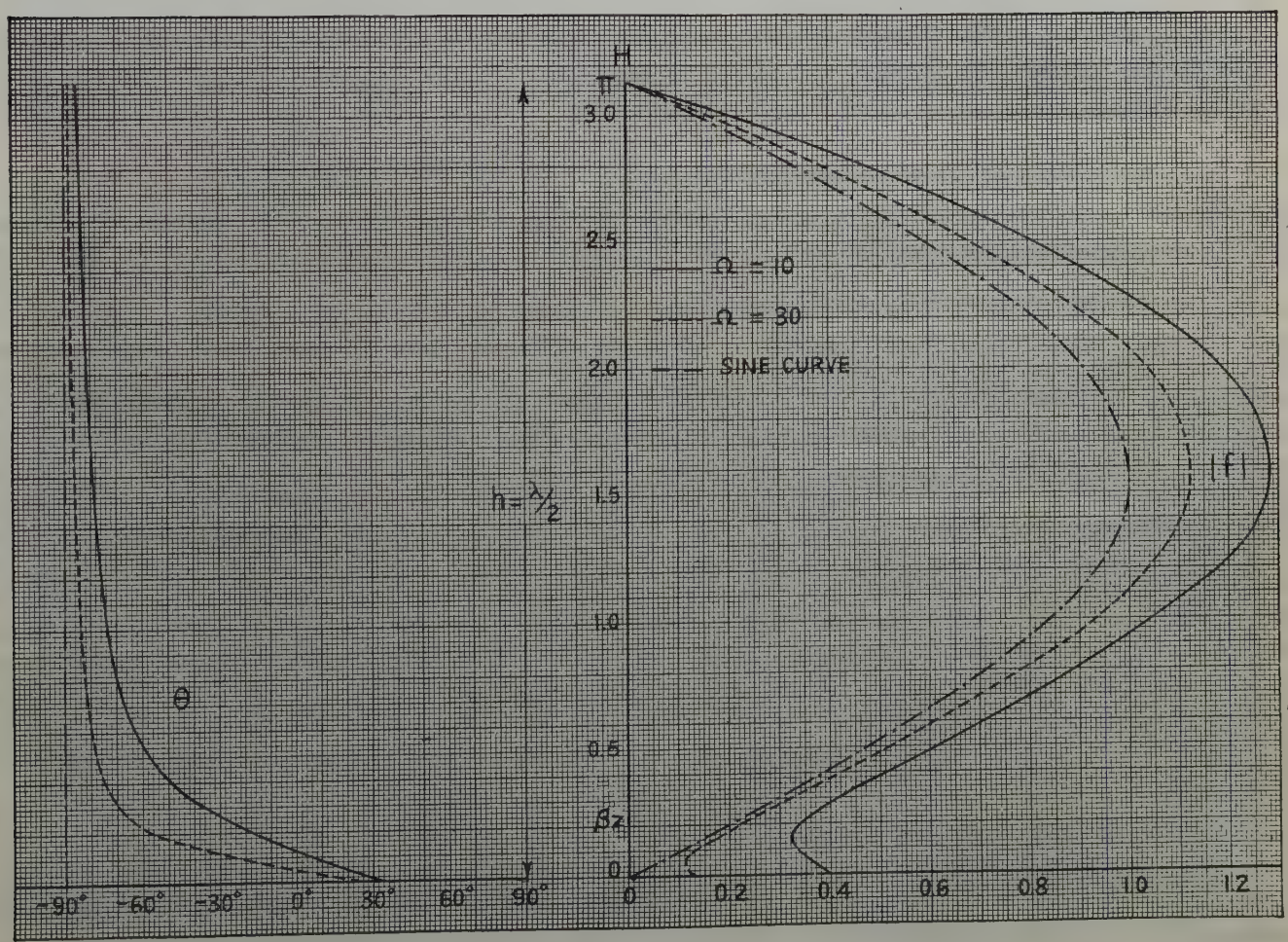

Fig. 16—The amplitude function $|f|$ and the phase function θ for $H=\pi$.

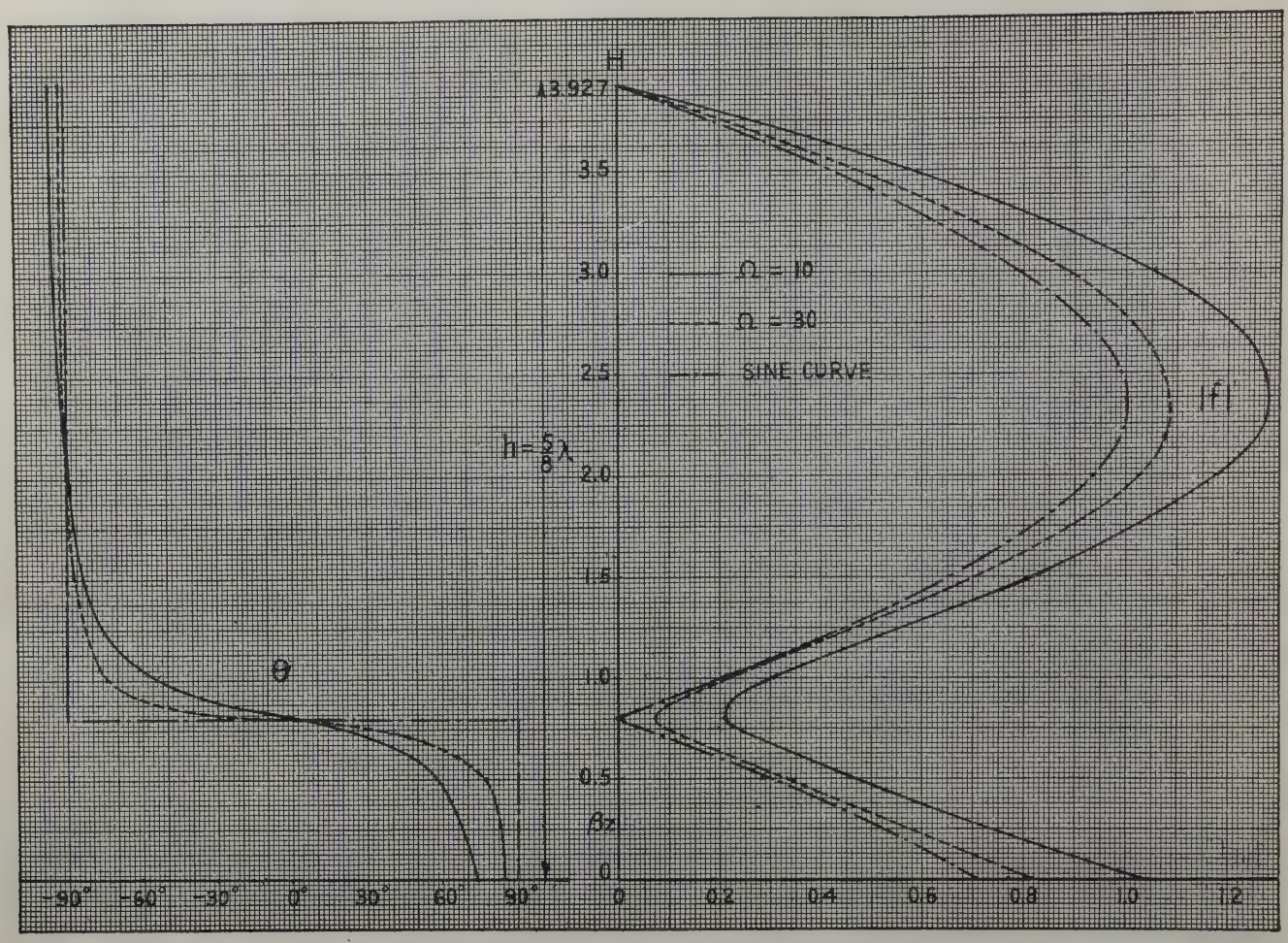

Fig. 17—The amplitude function $|f|$ and the phase function θ for $H=5\pi/4$.

and slightly greater length h' in the form $\beta h'=H'$ for H' in (70), and adjusting h' so that $\sin(H'-\beta|z|)/(\sin H')$ crosses the axis at or near the point where f' crosses it. $(H'-H$ should be approximately equal to $\pi/2-H_r$ where H_r is the value of H producing antiresonance for the particular choice of Ω. It may be obtained from Fig. 12 or equation (29b) in footnote reference 4.) At antiresonance, both B_{00} and $\sin H'$ must vanish. The indeterminate form $(V_0{}^e B_{00})/(\sin H')$ must then be replaced by the maximum value of I_z'. This always occurs near $z=h-(\lambda/4)$.

For purposes of calculating electromagnetic fields due to antennas of practically encountered thicknesses the distribution function (70) is a very much better approximation than the form (67) with (68) which is unsatisfactory over most of the range. (It is to be noted that at $H=n\pi/2$ with n odd (70) reduces exactly to (67).) The application of (70) in computing electromagnetic fields is reserved for a later paper.

Radiation Resistance Referred to Maximum Current

A common method of estimating the total power radiated from an antenna is to integrate the Poynting vector over a spherical surface in the far zone of the antenna. The calculation of the Poynting vector from the electric and magnetic fields is based on the assumption that the distribution of current has the simple sinusoidal form which is strictly accurate only for an indefinitely thin antenna. The total power so computed is then divided by the square of the maximum current (at $\lambda/4$ from the end of the antenna) to obtain the radiation resistance referred to maximum current. The curve marked $\Omega=\infty$ in Fig. 21 is that obtained and commonly reproduced for the sinusoidal distribution of current. Because the distribution of current in practical antennas is never exactly sinusoidal, even for $h=\lambda/4$ where the approximation is best, the radiation resistance so obtained is not accurate for them. Its correct value (neglecting power consumed in heating the antenna which is less than 3 per cent for copper antennas) can be determined as follows: Let the radiation resistance referred to maximum current be defined by

$$R_m{}^e = \frac{P_0}{|I_m|^2}. \tag{71}$$

Here P_0 is the power supplied to the antenna at its input terminals and, neglecting power consumed in heating the conductor, also the power radiated.

$$|I_m|^2 = [\sqrt{I_z'^2 + I_z''^2}]_{\max}. \tag{72}$$

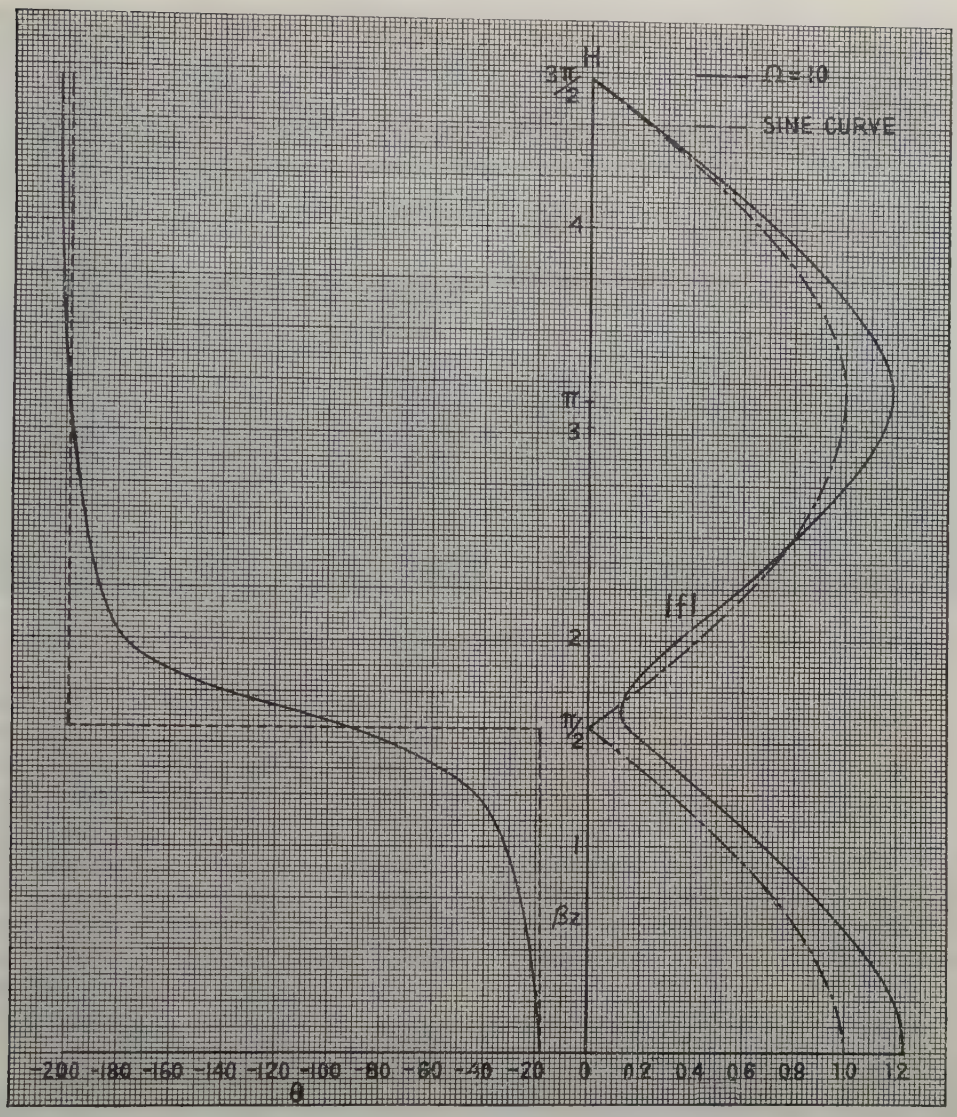

Fig. 18—The amplitude function $|f|$ and the phase function θ for $H=3\pi/2$.

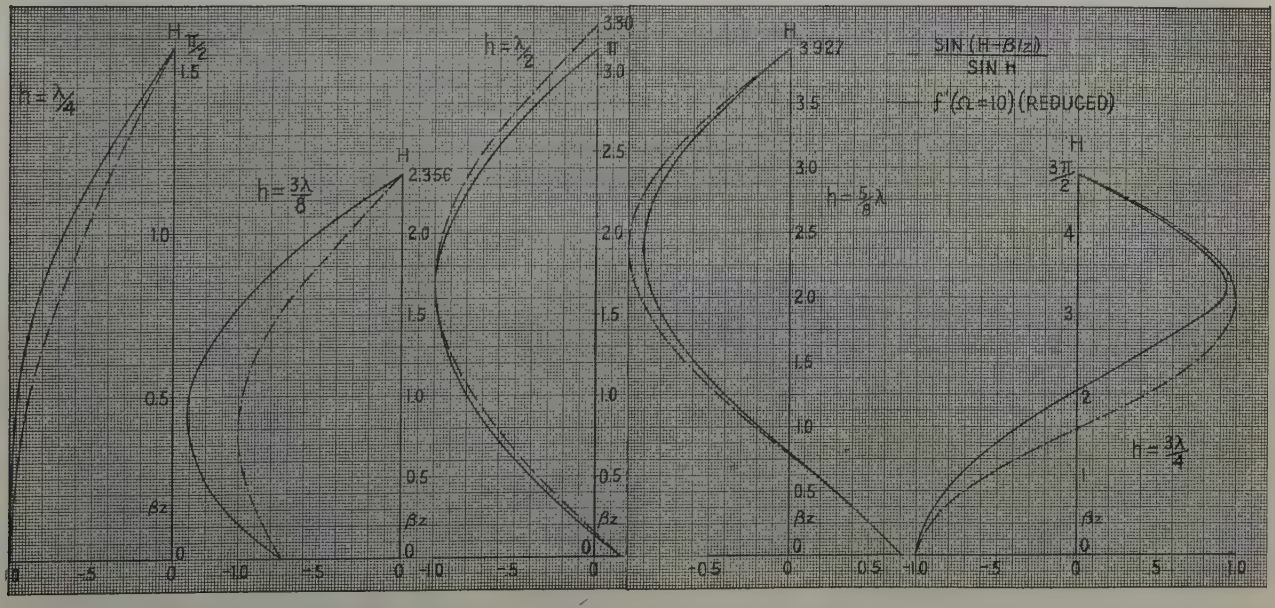

Fig. 19—The function sin $(H-\beta|z|)/\sin H$ compared with the distribution function f' reduced to the same value at $z=0$.

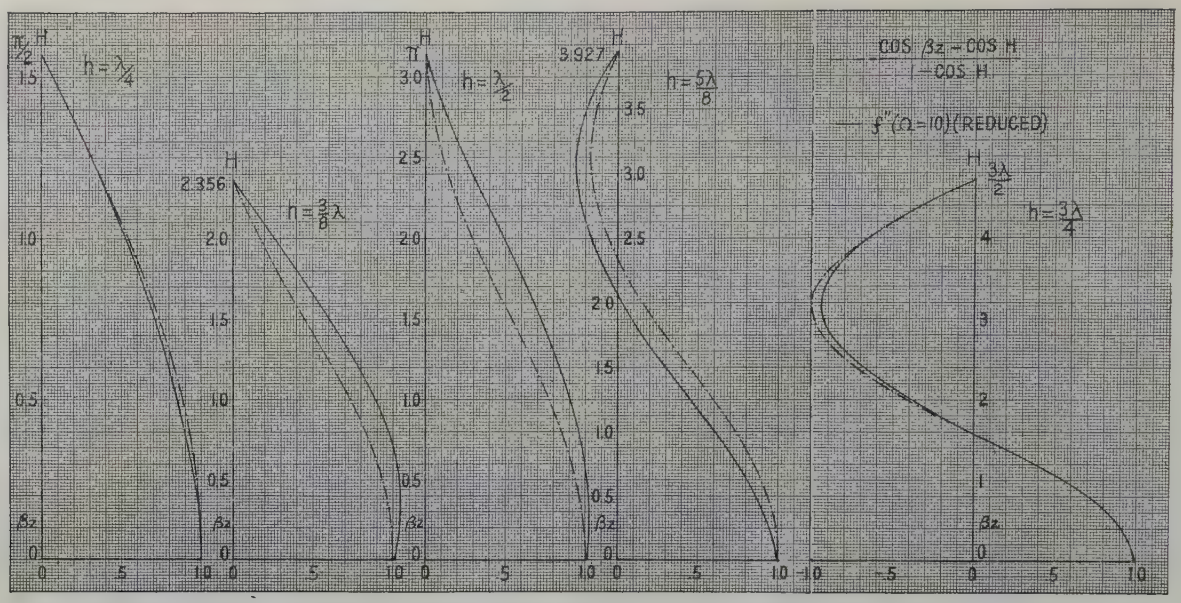

Fig. 20—The function $(\cos \beta z - \cos H)/(1-\cos H)$ compared with the distribution function f'' reduced to the same value at $z=0$.

Since
$$P_0 = |I_0|^2 R_{00} \tag{73}$$
one has
$$R_m^e = \left|\frac{I_0}{I_m}\right|^2 R_{00}. \tag{74}$$

Since $|I_0|$ and $|I_m|$ are readily determined from Figs. 13 to 18 and R_{00} may be computed directly from (11a), using Table I in footnote reference 4 for the particular value of Ω, R_m^e may be determined. The values so computed for the several lengths for which current data are available are shown plotted in small circles in Fig. 21 for a relatively thin antenna ($\Omega=30$) and a moderately thick antenna ($\Omega=10$). Although the number of available points is not sufficient to determine accurately the resulting curves near their maxima and minima, the general shape and position relative to the familiar curve for the simple sine distribution ($\Omega=\infty$) are correctly given. It is clear that the curve based on the sine distribution is not at all a good approximation for even moderately thick antennas except for lengths with H near $H=n\pi/2$ with n odd where the sinusoidal distribution is least in error. Even here it may be in error by as much as 50 per cent for very thick antennas. Actually R_m^e is completely unnecessary if the input impedance of an antenna is known.

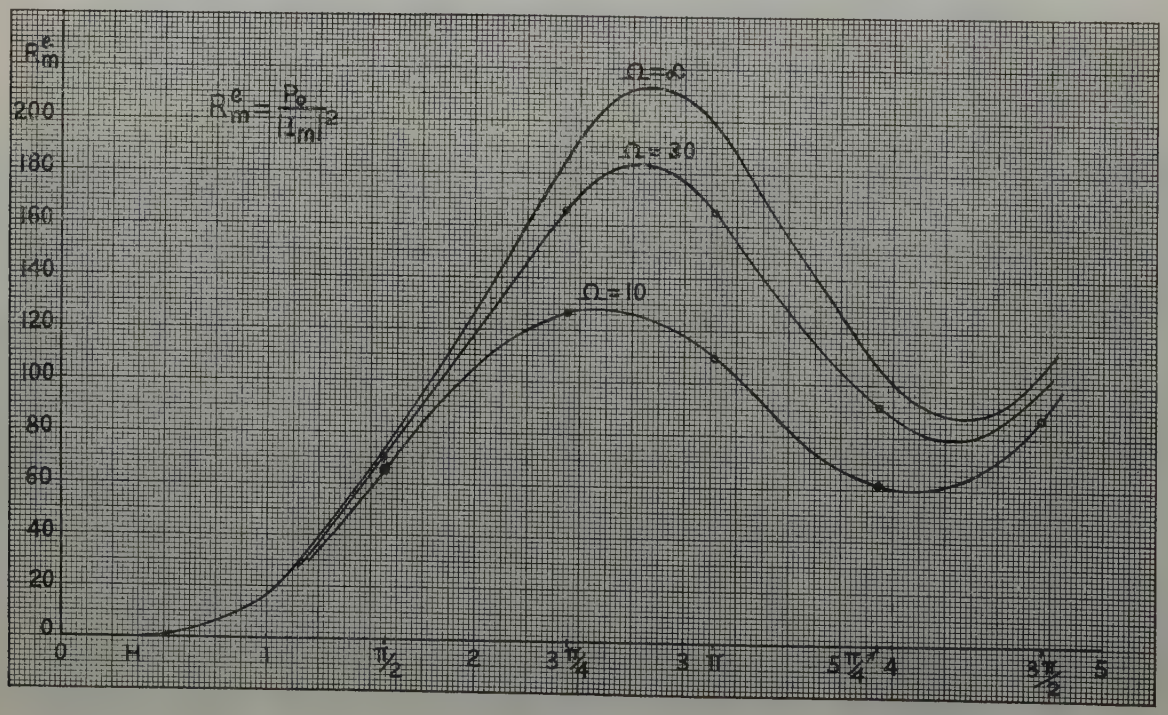

Fig. 21—The radiation resistance R_m^e referred to maximum current I_m. The curve marked $\Omega=\infty$ is accurately computed throughout. The curves marked $\Omega=10$ and $\Omega=30$ are estimated using only the insufficient number of accurately computed points shown by small circles.

Appendix I

The complex amplitude of the vector potential defined by (5) satisfies the Helmholtz equation

$$\nabla^2 A + \beta^2 A = i\Pi. \tag{75}$$

Here i is the volume density of current flowing, in this case, in the antenna. The Helmholtz integral which satisfies (75) is

$$A = \frac{\Pi}{4\pi}\int_\tau \frac{i'}{R_1} e^{-i\beta R_1} d\tau'. \tag{76}$$

Here τ is the volume of the cylindrical antenna, $d\tau'$ is an element of τ, i' is the current density at $d\tau'$, and R_1 is the distance between the point (r, θ, z) where A is computed and the element $d\tau'$ at (r', θ', z'). Since the radial component A_r can be due to i_r only, the inequality (8a) is readily verified. The z component is

$$A_z = \frac{\Pi}{4\pi}\int_\tau \frac{i_z'}{R_1} e^{-i\beta R_1} d\tau'. \tag{77}$$

It is to be proved that A_z evaluated from (22) on the cylindrical surface of the antenna differs by a negligible amount from A_z computed from the exact formula (75). I_z' in (22) is defined by

$$I_z' = \int_0^a \int_0^{2\pi} i_z' r dr d\theta = 2\pi \int_0^a i_z' r dr \tag{78}$$

since rotational symmetry may be assumed. Thus it must be shown that the following difference is vanishingly small.

$$D = \int_{-h}^{+h}\int_0^a \int_0^{2\pi} \frac{i_z'}{R_1} e^{-i\beta R_1} r d\theta' dr' dz' - \int_{-h}^{+h} \frac{I_z'}{R} e^{-i\beta R} dz'. \tag{79}$$

Here R is the distance from any point outside the antenna and not near the end faces where A is to be calculated, in particular a point on its cylindrical surface, to the element dz' on the *axis* of the antenna. R_1 is the distance from the same outside point to an element $r d\theta' dr' dz'$ in a cross section of the antenna at the center of which dz' is defined. This is illustrated in Fig. 22.

Because of rotational symmetry i_z' is independent of θ' and may be removed outside the sign of integration with respect to θ'. Thus with (78)

$$D = \int_{-h}^{+h} dz' \int_0^a i_z' r dr' \int_0^{2\pi} \left[\frac{e^{-i\beta R_1}}{R_1} - \frac{e^{-i\beta R}}{R}\right] d\theta'. \tag{80a}$$

It follows directly from (3) or from Fig. 22 that R_1 can differ from R at most by magnitudes of the order of the radius a of the cylinder. If R_1 and R are large compared with a, they will differ from each other by a negligible amount and the difference in the brackets in (6) will be vanishingly small. Accordingly, significant contributions to D for points on the surface of the antenna where $r = a$ can come only from that part of the integration with respect to z' for which $(z - z')$ is not large compared with a, i.e., from sections of the surface which are very close to the circumference at z'. It follows that the distribution of current at more distant points and even the length of the antenna can make no difference in the integral except at points very near the ends. Thus one may integrate from $-\infty$ to $+\infty$ and assume i_z' independent of the integration with respect to z'. Furthermore, since significant contributions

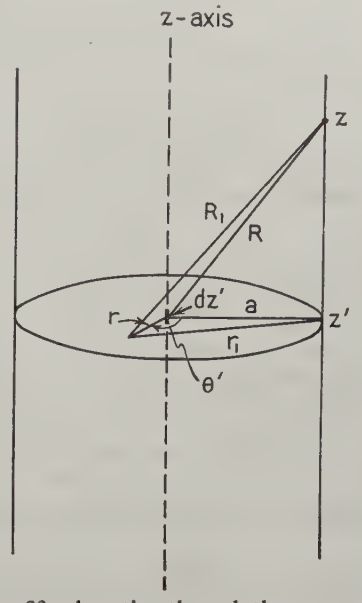

Fig. 22—A section through the antenna.

are obtained only for distances comparable in magnitude with a, and since with (3) $e^{-j\beta a} \doteq 1$, the two exponentials may be set equal to unity in the range of significant contributions. This leaves

$$D = \int_0^a i_z' r dr' \int_0^{2\pi} d\theta' \int_{-\infty}^{+\infty} \left(\frac{1}{R_1} - \frac{1}{R}\right) dz'. \tag{80b}$$

The integral with respect to z' can be integrated directly into $\ln (a/r_1)^2$ with r_1 indicated in Fig. 22. If this is integrated with respect to θ' the integral vanishes. Thus D is entirely negligible except at points within distances of the ends of the antenna comparable with a. The contributions of these short sections were made negligible by imposing (3a).

(It is interesting to note that the rigorous derivation of the transmission-line equations for parallel wires from fundamental electromagnetic theory depends upon exactly the same demonstration in that (22) is assumed valid on the surface of each parallel wire.)

Appendix II

In order to derive the solution (39) from the integral equation (38) it is convenient to introduce the following shorthand notation:

$$F_0(z) \equiv \cos \beta z; \qquad G_0(z) \equiv \sin \beta |z| \tag{81}$$

$$F_{0z} \equiv F_0(z) - F_0(h); \qquad G_{0z} \equiv G_0(z) - G_0(h). \tag{82}$$

The first brace in (38) will be denoted by $(I_z)_0$. Using the above notation it is

$$(I_z)_0 = \frac{-j4\pi}{\Omega R_a}\left\{C_1 F_{0z} - \tfrac{1}{2} V_0^e G_{0z}\right\}. \tag{83}$$

It may be regarded as a zeroth order approximation for I_z. If it is substituted in the rest of the terms in (38), all

of which include the current as a factor, terms will be obtained which are multiplied by $1/\Omega^2$. They will be denoted by $(I_z')_0$. They are

$$(I_z')_0 = \frac{-j4\pi}{\Omega^2 R_c}\left\{C_1 F_{1z} - \tfrac{1}{2}V_0^e G_{1z}\right\} \tag{84}$$

with

$$F_{1z} \equiv F_1(z) - F_1(h); \quad G_{1z} \equiv G_1(z) - G_1(h) \tag{85}$$

and

$$F_1(z) = -F_{0z}\ln\left(1-\frac{z^2}{h^2}\right) + F_{0z}\delta - \int_{-h}^{+h}\frac{F_{0z}'e^{-j\beta R} - F_{0z}}{R}dz' \left\{-\frac{j4\pi z^i}{R_c}\int_0^z F_{0s}\sin\beta(z-s)ds\right\} \tag{86}$$

$$F_1(h) = -\left\{\int_{-h}^{+h}F_{0z}'\frac{e^{-j\beta R_h}}{R_h}dz' - \frac{j4\pi z^i}{R_c}\int_0^h F_{0s}\sin\beta(h-s)ds\right\}. \tag{87}$$

$G_1(z)$ is exactly like $F_1(z)$ with G written for F throughout. (88)

$G_1(h)$ is exactly like $F_1(h)$ with G written for F throughout. (89)

The first-order approximation for I_z is now given by

$$(I_z)_1 \equiv (I_z)_0 + (I_z')_0 = \frac{-j4\pi}{\Omega R_c}\left\{C_1\left[F_{0z}+\frac{F_{1z}}{\Omega}\right] + \tfrac{1}{2}V_0^e\left[G_{0z}+\frac{G_{1z}}{\Omega}\right]\right\}. \tag{90}$$

If this expression is substituted for I_z on the right in (38) a second-order approximation may be obtained. This may be substituted back in (38) to obtain a third-order solution. The process may be continued indefinitely to obtain a series solution of the form

$$I_z = \frac{-j4\pi}{\Omega R_c}\left\{C_1\left[F_{0z} - \frac{F_{1z}}{\Omega} + \frac{F_{2z}}{\Omega^2} + \cdots\right] + \tfrac{1}{2}V_0^e\left[G_{0z} + \frac{G_{1z}}{\Omega} + \frac{G_{2z}}{\Omega^2} + \cdots\right]\right\}. \tag{91}$$

The constant of integration C_1, can now be evaluated directly by substituting the solution (91) for I_z in (36). If this is done one has

$$0 = \frac{-j4\pi}{\Omega R_c}\left\{C_1\left[F_0(h) + \frac{F_1(h)}{\Omega} + \cdots\right] + \tfrac{1}{2}V_0^e\left[G_0(h) + \frac{G_1(h)}{\Omega} + \cdots\right]\right\}. \tag{92}$$

This can be solved for C_1 as follows:

$$C_1 = -\tfrac{1}{2}V_0^e\left\{\frac{G_0(h) + (1/\Omega)G_1(h) + \cdots}{F_0(h) + (1/\Omega)F_1(h) + \cdots}\right\}. \tag{93}$$

If this is inserted in (91) one has the solution for I_z. It is

$$I_z = \frac{j2\pi V_0^e}{\Omega R_c}\left\{\frac{[F_{0z}+F_{1z}/\Omega+\cdots][G_0(h)+G_1(h)/\Omega+\cdots] - [G_{0z}+G_{1z}/\Omega+\cdots][F_0(h)+F_1(h)/\Omega+\cdots]}{F_0(h)+F_1(h)/\Omega+\cdots}\right\}. \tag{94}$$

After rearranging using (82) and (85) one has precisely (39).

In carrying out the evaluation of the function $F_1(z)$, $F_1(h)$, $G_1(z)$, and $G_1(h)$ as defined in (86) and (87), advantage can be taken of the fact that a^2 is negligible compared with $(h-z)^2$ and $(h+z)^2$ except for points very near the ends of the antenna. At the ends errors as large as 50 per cent are involved. However, since the current necessarily vanishes at the ends, the distribution of current is actually not significantly affected. At most the current within distances of the ends comparable with the radius a may be in error by an appreciable amount, but this error becomes negligible at distances of three or four times the radius from the end. Actually in computing the current, points need not be taken within distances of the ends comparable with the radius a and a curve connecting points at distances of $5a$ or more from the ends to zero values at the ends must give the correct distribution. Accordingly the term in δ may be neglected and one may write

$$R \doteq |z' - z| \tag{95}$$

in the integrals. It is especially important to note that the approximations here introduced are extremely good for the current at all points except near the ends where it is known to vanish. In particular, the input current, and hence the input impedance, is in no way affected if (3) is fulfilled.

If one makes use of (95) all integrals in (86) to (89) are readily evaluated without further approximations in terms of trigonometric functions or the integral functions defined below.

$$\int_a^b \frac{1-\cos u}{u}du = \overline{\mathrm{Ci}}\,(b) - \overline{\mathrm{Ci}}\,(a) \tag{96}$$

$$\int_a^b \frac{\sin u}{u}du = \mathrm{Si}\,(b) - \mathrm{Si}\,(a). \tag{97}$$

The final forms are

$$\begin{aligned}F_1(z) = &-(\cos\beta z - \cos\beta h)\ln(1-z^2/h^2)\\ &+\tfrac{1}{2}\cos\beta z[\overline{\mathrm{Ci}}\,2\beta(h+z)+\overline{\mathrm{Ci}}\,2\beta(h-z)\\ &+j\,\mathrm{Si}\,2\beta(h+z)+j\,\mathrm{Si}\,2\beta(h-z)]\\ &-\tfrac{1}{2}\sin\beta z[\mathrm{Si}\,2\beta(h+z)-\mathrm{Si}\,2\beta(h-z)\\ &-j\,\overline{\mathrm{Ci}}\,2\beta(h+z)+j\,\overline{\mathrm{Ci}}\,2\beta(h-z)]\\ &-\cos\beta h[\overline{\mathrm{Ci}}\,\beta(h+z)+\overline{\mathrm{Ci}}\,\beta(h-z)\\ &+j\,\mathrm{Si}\,\beta(h+z)+j\,\mathrm{Si}\,\beta(h-z)]\\ &+j\frac{4\pi z^i h}{R_c}\left[\frac{z}{2h}\sin\beta z - \frac{\cos\beta h}{\beta h}(1-\cos\beta z)\right]\end{aligned} \tag{98}$$

$$\begin{aligned}G_1(z) = &-(\sin\beta|z| - \sin\beta h)\ln(1-z^2/h^2)\\ &-\tfrac{1}{2}\cos\beta z[\mathrm{Si}\,2\beta(h+z)+\mathrm{Si}\,2\beta(h-z)\\ &-2\,\mathrm{Si}\,(2\beta|z|)-j\,\overline{\mathrm{Ci}}\,2\beta(h+z)\\ &-j\,\overline{\mathrm{Ci}}\,2\beta(h-z)+2j\,\overline{\mathrm{Ci}}\,(2\beta z)]\\ &-\tfrac{1}{2}\sin\beta z[\overline{\mathrm{Ci}}\,2\beta(h+z)-\overline{\mathrm{Ci}}\,2\beta(h-z)\\ &+j\,\mathrm{Si}\,2\beta(h+z)-j\,\mathrm{Si}\,2\beta(h-z)-2j\,\mathrm{Si}\,(2\beta z)]\end{aligned}$$

$$-\tfrac{1}{2}\sin\beta|z|\left[4\ln\frac{|z|}{h+|z|}-2\,\overline{\mathrm{Ci}}\,(2\beta z)\right]$$
$$-\sin\beta h[\overline{\mathrm{Ci}}\,\beta(h+z)+\overline{\mathrm{Ci}}\,\beta(h-z)$$
$$+j\,\mathrm{Si}\,\beta(h+z)+j\,\mathrm{Si}\,\beta(h-z)]$$
$$+j\frac{4\pi z^2 h}{R_c}\left[\frac{\sin\beta|z|}{2\beta h}-\frac{|z|\cos\beta z}{2h}\right.$$
$$\left.-\frac{\sin\beta h}{\beta h}(1-\cos\beta z)\right] \qquad (99)$$

$$F_1(h)=\tfrac{1}{2}\cos\beta h[\overline{\mathrm{Ci}}\,(4\beta h)+j\,\mathrm{Si}\,(4\beta h)]$$
$$-\tfrac{1}{2}\sin\beta h[\mathrm{Si}\,(4\beta h)-j\,\overline{\mathrm{Ci}}\,(4\beta h)]$$
$$-\cos\beta h[\overline{\mathrm{Ci}}\,(2\beta h)+j\,\mathrm{Si}\,(2\beta h)]$$
$$+j\frac{4\pi z^2 h}{R_c}\left[\frac{\sin\beta h}{2}-\frac{\cos\beta h}{\beta h}(1-\cos\beta h)\right] \qquad (100)$$

$$G_1(h)=-\tfrac{1}{2}\cos\beta h[\mathrm{Si}\,(4\beta h)-2\,\mathrm{Si}\,(2\beta h)$$
$$-j\,\overline{\mathrm{Ci}}\,(4\beta h)+2j\,\overline{\mathrm{Ci}}\,(2\beta h)]$$
$$-\tfrac{1}{2}\sin\beta h[\overline{\mathrm{Ci}}\,(4\beta h)+j\,\mathrm{Si}\,(4\beta h)$$
$$-2j\,\mathrm{Si}\,(2\beta h)-2\,\overline{\mathrm{Ci}}\,(2\beta h)-4\ln 2]$$
$$-\sin\beta h[\overline{\mathrm{Ci}}\,(2\beta h)+j\,\mathrm{Si}\,(2\beta h)]$$
$$-j\frac{4\pi z^2 h}{R_c}\left[\frac{\sin\beta h}{2\beta h}+\tfrac{1}{2}\cos\beta h-\frac{\sin\beta h\cos\beta h}{\beta h}\right] \qquad (101)$$

The terms in z^2 were neglected in computing the curves of Figs. 2 to 6. They are entirely negligible if the antenna is a good conductor.

It has been planned to present in the PROCEEDINGS of the I.R.E. instructional material of timely interest. This procedure was instituted some time ago, and here continues by the publication, in successive issues of the PROCEEDINGS, of a series of coordinated parts, together entitled "Some Aspects of Radio Reception at Ultra-High Frequency" by Messrs. E. W. Herold and L. Malter. Parts IV and V of the five parts are here presented. Each Part is preceded by its own related summary. *The Editor*

Some Aspects of Radio Reception at Ultra-High Frequency*

E. W. HEROLD†, MEMBER, I.R.E., AND L. MALTER‡, ASSOCIATE, I.R.E.

PART IV. GENERAL SUPERHETERODYNE CONSIDERATIONS AT ULTRA-HIGH FREQUENCIES

L. MALTER‡

Summary—**This paper presents a general survey of the problems encountered in the mixer or converter stage of superheterodyne receivers, particularly at ultra-high frequencies. The application of a strong local-oscillator voltage causes a periodic variation of the signal-electrode transconductance as a consequence of which intermediate-frequency-current components appear in the output circuit when a signal is also impressed upon the signal electrode. It is demonstrated that intermediate-frequency-current components are present in the output, which differ from the signal frequency by integral multiples of the local-oscillator frequency, if the Fourier analysis of the signal-electrode transconductance contains components which are integral multiples of the local-oscillator frequency. Methods of determining the conversion transconductance for so-called fundamental and harmonic conversion are given.**

It is shown that the noise output and input loading of a mixer stage are given by averaging these quantities over a local-oscillator cycle. A discussion of mixer gain is included, with a demonstration that the gain of a mixer stage is given approximately by the product of the conversion transconductance and the impedance of the output circuit (for high-output-impedance tubes).

Considerations regarding image rejection and the undesirability of radiation of oscillator power lead to the conclusion that high intermediate frequencies are desirable.

An extended discussion of whether to use an amplifier or mixer stage in the first stage of a superheterodyne receiver is included. If the received signal is strong, one should convert immediately, unless image rejection or the prevention of oscillator radiation necessitate the use of radio-frequency stages. If the received signal is weak, an amplifier stage should be used below a certain frequency and a mixer above, the transition frequency depending upon the characteristics of the tubes available and the bandwidth required. In general the transition frequency occurs at the point where available tubes will no longer give appreciable radio-frequency gain for the bandwidth required.

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I. INTRODUCTION

IN PART II of this series we concerned ourselves primarily with the case wherein the signal voltages applied to the circuits and tubes of a receiver are so low in amplitude that the tubes can be considered as linear devices, wherein the output voltage or current is proportional to the signal-electrode voltage. This case will be recognized as being precisely that of the linear amplifier.

It is frequently convenient, however, to make use of the superheterodyne principle in receivers. In receivers of this type, the incoming signal is combined with a locally produced oscillation of different frequency to produce a third signal at a frequency referred to as the intermediate frequency, which is related to both the frequencies of the incoming signal and the locally produced oscillation. It is an essential characteristic of any

electrical device, wherein the simultaneous application of oscillating voltages results in the production of one or more oscillating quantities whose frequencies differ from those of any of the impressed quantities, that the device be *nonlinear*, i.e., that the relation between the output and input (i.e., impressed) variables be of such a nature that it cannot be reduced to a form wherein the variables appear as first-power terms, or that the nature of the device be such that, even though it be linear as regards the application of a single voltage, the simultaneous application of two voltages results in the appearance in the output of a quantity (e.g., a current change) which is dependent upon the product of the two impressed voltages. An example of such a device is a multigrid tube wherein two voltages may be impressed simultaneously on different grids. In this case the change in output current will be related to the changes in the two grid voltages by means of a relation of the form

$$i_p = b_0 + b_1e_1 + b_2e_2 + b_{12}e_1e_2 \qquad (1)$$

where the b's are constants, i_p is the change in plate current, and e_1 and e_2 are the changes in potential of the two grids. Since, in a superheterodyne receiver, the simultaneous application of two voltages of different frequencies results in the production of the intermediate-frequency signal whose frequency differs from those of the two original signals, the superheterodyne receiver must contain a device which has one of the two types of characteristics described above. This device is generally a tube which may (among others) be a diode, triode, or multigrid tube.

In Fig. 1 of Part V (which follows Part IV in this issue) there is shown an incoming signal and locally produced oscillation fed into a diode with the resultant production of intermediate-frequency output. The diode as a device for producing intermediate frequencies is sufficiently important and different from other tubes designed for the same purpose, so as to merit special treatment. It will form the subject matter of Part V of this series.

In the case of triodes or pentodes various modes of introducing the incoming signal and locally produced oscillation are possible. Thus, they may both be impressed between control-grid and cathode, as is illustrated in Fig. 1, or the incoming signal may be impressed between the control grid and ground, while the locally produced oscillation is impressed between cathode and ground. In general the incoming signal and the locally produced oscillations need not be impressed upon the same electrodes. Throughout the remainder of this paper the term signal electrode will refer to the control electrode upon which the incoming signal is impressed.

For many superheterodyne applications at frequencies below 30 megacycles or so, it has been found convenient to make use of multitube grids. In this case, in addition to the possibility of impressing both voltages on the same electrode, one can also have the local-oscillator voltage applied to an electrode which precedes the signal electrode (inner-grid injection) or the converse (outer-grid injection). Since tubes of this type do not find much application at ultra-high frequencies, their consideration is here terminated, except to indicate that the interested reader may find a more extended treatment in the literature.[1]

Before launching into a discussion of the mechanism of mixer action, it may be to the point to outline briefly the outstanding reasons for the use of superheterodyne receivers at ultra-high frequencies:

1. The tuned circuits in intermediate-frequency amplifiers are fixed in frequency, whereas in radio-frequency amplifiers they require tuning to the individual signal. If the receiver is to cover an extended frequency range, the tuning problem for the radio-frequency amplifier may be a serious one.
2. It is often possible to secure a higher gain per stage at intermediate frequencies than at radio frequencies.
3. Better control of frequency response can generally be achieved at intermediate frequencies, particularly if the receiver is to operate over an extended frequency range.
4. It may be possible to achieve a higher signal-to-noise ratio with a superheterodyne type receiver than with a radio-frequency amplifier type.

II. The Mechanism of Mixer Action

The mechanism whereby the combination of the incoming signal and the locally produced oscillation serves to produce an intermediate frequency is referred to as "mixing action." The locally produced oscillation may either be generated in the same tube as that in which the mixing action occurs in which case this tube that serves a double function, is referred to as a converter, or the locally produced oscillation may be produced in a separate tube in which case the tube in which the mixing action occurs is referred to as a mixer.

The fact that a nonlinear device can be used for the production of an intermediate-frequency signal, is readily demonstrated by means of a simple illustration. In Fig. 1 there is shown a triode with input circuits upon

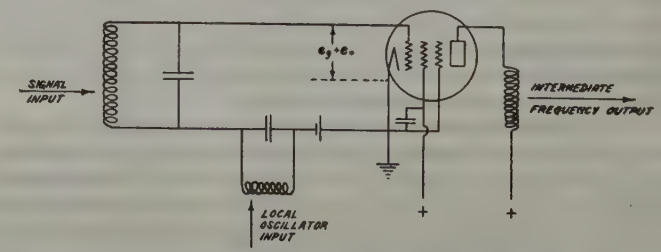

Fig. 1—Circuit of pentode mixer stage wherein incoming signal and local-oscillator voltage are both applied between control grid and cathode.

which are impressed an incoming signal and a locally produced oscillation. These will result respectively in the impression upon the grid of the triode of two

[1] E. W. Herold, "The operation of frequency converters and mixers for superheterodyne reception," Proc. I.R.E., vol. 30, pp. 84–103; February, 1942.

oscillating voltages e_g and e_0. The presence of these voltages will cause a change in plate current of an amount i_p. Since e_g, e_0, and i_p are variations away from quiescent values, we can set (by the application of Taylor's theorem)

$$i_p = c_0 + c_1 e + c_2 e^2 + c_3 e^3 + \cdots \tag{2}$$

where $e = e_g + e_0$ and the c's are constants which depend upon the tube and circuit characteristics. For the amplifier case, e is so small that the terms of higher power than the first may be neglected. Equation (2) then becomes

$$i_p = c_0 + c_1 e. \tag{2a}$$

From (2a) it may be seen that i_p will contain as components, terms whose frequency is the same as those present in e. For the generation of current components of frequencies different from those of e_g and e_0 it is essential that e be sufficiently great so that some of the terms in (2) of the second or higher degree become of importance. Since e_s is, in general, comparatively small, e_0 must be comparatively large in order that the relation between i_p and e be nonlinear. Actually, while e_s is generally of the order of microvolts or millivolts, e_0 is usually measured in volts.

To see that a nonlinear relationship of the form of (1) results in the production of terms of frequencies different from those of e_g and e_0, let us consider the simple case wherein the terms of powers higher than the second may be neglected. Then

$$i_p = c_0 + c_1(e_g + e_0) + c_2(e_g + e_0)^2. \tag{3}$$

Let
$$e_s = E_s \sin \omega_s t \tag{4}$$
$$e_0 = E_0 \sin \omega_0 t. \tag{5}$$

Then
$$\begin{aligned} i_p &= c_0 + c_1 E_s \sin \omega_s t + c_1 E_0 \sin \omega_0 t \\ &\quad + c_2 E_s^2 \sin^2 \omega_s t + 2c_2 E_s E_0 \sin \omega_s t \sin \omega_0 t \\ &\quad + c_2 E_0^2 \sin^2 \omega_0 t \end{aligned} \tag{6}$$

$$\begin{aligned} &= c_0 + c_1 E_s \sin \omega_s t + c_1 E_0 \sin \omega_0 t + (c_2 E_s^2)/2 \\ &\quad + (c_2/2) E_s^2 \sin 2\omega_s t + c_2 E_s E_0 \cos (\omega_s - \omega_0) t \\ &\quad - c_2 E_s E_0 \cos (\omega_s + \omega_0) t + (c_2 E_0^2)/2 \\ &\quad + [(c_2 E_0^2)/2] \sin 2\omega_0 t. \end{aligned} \tag{7}$$

Thus, in addition to terms of angular frequency ω_s and ω_0, there appear in the plate current terms of angular frequency $2\omega_0$, $2\omega_s$, $|\omega_s - \omega_0|$, and $|\omega_s + \omega_0|$. If the output circuit in Fig. 2 is resonant to angular frequency $|\omega_s - \omega_0|$, voltage at this angular frequency will be developed across the circuit. This may then be applied to later intermediate-frequency amplifier stages. For the case just treated the intermediate frequency may be of angular frequency $|\omega_s - \omega_0|$ or $|\omega_s + \omega_0|$. For the general case as represented by (1), the intermediate-frequency term will be of the form $|n_1\omega_s \pm n_2\omega_0|$ where n_1 and n_2 are integers. The output circuit is made to resonate at the particular angular frequency which is chosen for intermediate-frequency amplification. The most common choice is the one wherein $n_1 = n_2 = 1$ and $\omega_i = |\omega_s - \omega_0|$, where ω_i is the angular frequency of the intermediate-frequency signal.

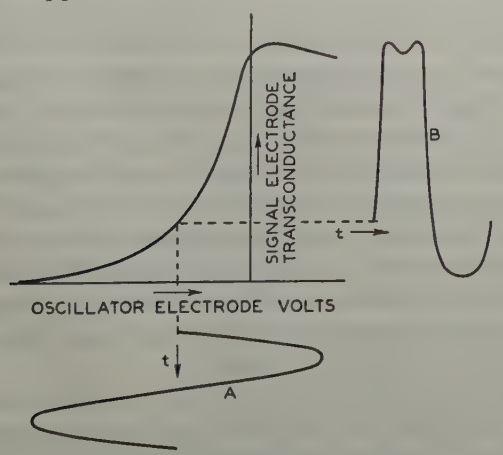

Fig. 2—Signal-electrode transconductance versus oscillator-electrode voltage for a typical mixer tube. The applied oscillator voltage is shown at *A* and the resultant time variation of the transconductance at *B*.

The preceding treatment, while of value in portraying the mechanism whereby the mixing action is brought about, is unsatisfactory for numerical computations, since, for normal tube characteristics, and for large oscillator voltages, it leads to exceedingly complex expressions and very laborious computations. A more elegant mode of attack upon the problem of computing mixer action has been described by Herold.[1] Before entering into this it is necessary to introduce the quantity known as conversion transconductance which is defined as the ratio of the intermediate-frequency output current to the signal-electrode input voltage.

As was pointed out above, one mode of operation is that in which the local-oscillator signal should be sufficiently large so as to result in nonlinear operation of the mixer or converter. While the nonlinearity was defined in terms of the relation of output-current-to-signal-electrode voltage, it is obvious that the nonlinearity must at the same time extend to the signal-electrode-to-plate transconductance. As a consequence, due to the presence of the "large" local-oscillator voltage, the instantaneous signal-electrode transconductance may be considered as varying periodically at local-oscillator frequency, and may thus be expressed in the form of a Fourier series

$$g_m = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \cdots \tag{8}$$

where the a's are constants which depend upon the tube characteristic, the quiescent or operating point, and upon the magnitude of the local-oscillator voltage. Since $\omega_0 = 2\pi f_0$, where f_0 is the local-oscillator frequency, the signal-electrode transconductance may be considered as being made up of an infinite number of components, the first being an average value a_0, the second a term of oscillator frequency, the third a term of twice oscillator frequency, etc. Now, simultaneous with the application of the local-oscillator voltage, let a signal voltage $E_s \sin \omega_s t$ be applied to the signal electrode. Then the plate current will be given by

$$i_p = g_m E_s \sin \omega_s t. \tag{9}$$

For g_m in (9) we substitute its value from (8) and obtain

$$i_p = a_0 E_s \sin \omega_s t + E_s \sum_{n=1}^{\infty} a_n \sin \omega_s t \cos n\omega_0 t \tag{10}$$

$$= a_0 E_s \sin \omega_s t + E_s/2 \sum_{n=1}^{\infty} a_n \sin (\omega_s + n\omega_0)t \quad (11)$$
$$+ E_s/2 \sum_{n=1}^{\infty} a_n \sin (\omega_s - n\omega_0)t.$$

The plate current thus contains components of angular frequency ω_s, $|\omega_s + n\omega_0|$, and $|\omega_s - n\omega_0|$. The first of these is one at signal frequency and does not concern us. The other terms are those which characterize the device as a mixer since they represent new frequency terms related to the signal and local-oscillator frequencies. It is customary to insert a tuned circuit in the output lead of the converter or mixer which is tuned to angular frequency $|\omega_s - \omega_0|$. The voltage developed across this circuit is then fed into the intermediate-frequency amplifier. While this mode of operation (referred to as fundamental operation, since the intermediate frequency is equal in absolute value to the difference between the signal frequency and the fundamental local-oscillator frequency) is generally the preferred one, in certain cases it is convenient to operate at one of the other possible intermediate frequencies for which $n \neq 1$; (referred to as harmonic operation). In these cases, the tuned output circuit is tuned to angular frequency $|\omega_s - n\omega_0|$. It should be noted that for harmonic operation it is not the oscillator voltage or tube current which must contain harmonic components, but the Fourier analysis of the signal-electrode transconductance, when the tube is operated as a mixer. Harmonic operation may be used when generation of a local-oscillator signal of sufficient power at fundamental frequency is difficult. However, in general, harmonic operation yields a lower signal-to-noise ratio than fundamental operation, and is thus avoided, if possible, where signal-to-noise ratio is of fundamental importance (as is the case in many ultra-high-frequency applications).

The conversion transconductance at the nth harmonic is given by

$$g_{c_n} = I(\omega_0 - n\omega_n)/E_s = a_n/2. \quad (12)$$

If one substitutes the value of the Fourier coefficient, there results

$$g_{c_n} = \frac{1}{2\pi} \int_0^{2\pi} g_m \cos n\omega_0 t d\omega_0 t. \quad (13)$$

When $n = 1$, the fundamental conversion transconductance is obtained.

The method of determining conversion transconductance can be made clear by means of an illustrative example. Let Fig. 2 represent the signal-electrode transconductance of a receiving tube as a function of oscillator-electrode voltage (it is assumed, as may be the case, that the oscillator voltage and signal voltage are not necessarily applied to the same electrode), and let A in the figure be the applied oscillator voltage (assumed to be sinusoidal). Then B is the resultant time variation of transconductance.

A Fourier analysis of B yields the desired conversion transconductance since for the nth harmonic mode of operation it is simply half the nth Fourier coefficient, as was shown above. If, as is usually the case, the oscillator voltage is sinusoidal in shape, it is possible to make use of some convenient formulas of sufficient accuracy for most purposes. Referring to Fig. 3, a sinusoidal oscillator voltage is assumed and a seven-point analysis at 30-degree intervals is made. Then,[1] the conversion transconductances for the fundamental and the first two modes of harmonic operation are

$$g_{c_1} = \tfrac{1}{12}[(g_7 - g_1) + (g_5 - g_3) + 1.73(g_6 - g_2)] \quad (14)$$
$$g_{c_2} = \tfrac{1}{12}[2g_4 + \tfrac{3}{4}(g_3 + g_5 - g_6 - g_2) - (g_7 + g_1)] \quad (15)$$
$$g_{c_3} = \tfrac{1}{12}[(g_7 - g_1) - 2(g_5 - g_3)]. \quad (16)$$

The values g_1, g_2, etc., are obtained from the transconductance characteristic of Fig. 3 by means of the 30-degree analysis there indicated. An examination of (14) for g_{c_1} indicates that maximum possible fundamental conversion transconductance, (excluding negative transconductances) occurs when g_1, g_2, and g_3 are zero and g_5, g_6, and g_7 are large. This is achieved by operating so that the transconductance is cut off over slightly less than half the oscillator cycle and with oscillator-voltage amplitude of such magnitude that the tube operates to somewhat beyond the point of maximum transconductance.

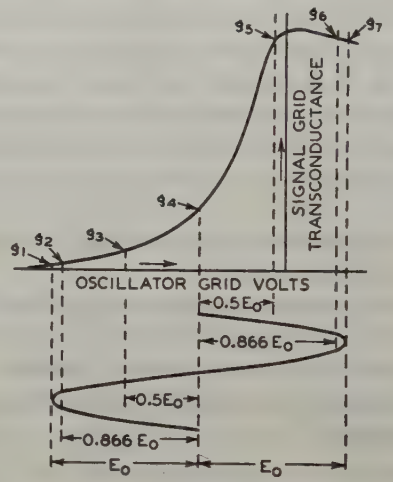

Fig. 3—Points used for 30-degree analysis of conversion transconductance.

In practical cases, wherein grid-controlled tubes are employed, the maximum possible fundamental conversion transconductance is given approximately[1] by 28 per cent of the maximum signal-grid-to-plate transconductance. The maximum attainable second-harmonic conversion transconductance is roughly half as great, and for third-harmonic conversion the maximum attainable transconductance is only about one third as great. Harmonic operation requires greater excitation, as a rule, than fundamental operation and the optimum operating point is differently located.

Thus an examination of the signal-electrode-to-plate transconductance curve quickly yields approximate values of the following quantities:

1. Optimum operating point for fundamental conversion.

2. Optimum local-oscillator excitation for fundamental conversion.

3. Conversion transconductances for fundamental, second harmonic, and third harmonic conversion.

III. Noise of Converter or Mixer Stage

As was shown in Parts II and III of this series, fluctuation noise is measured by its mean-square value. In most vacuum tubes, the important part of the fluctuation noise comes from the plate or anode current. Under static conditions or with very small signals the mean-square noise current may be written as $\overline{i_{p_n}^2}$. In the converter stage of a receiver, a large local oscillator voltage is usually applied to the converter or mixer so that $\overline{i_{p_n}^2}$ fluctuates periodically at local-oscillator frequency. The mean-square noise current at the intermediate frequency, which is the one of concern here, is then the average value of the fluctuations, i.e., the time average over the oscillator cycle,

$$\overline{i_{i-f}^2} = \frac{1}{2\pi}\int_0^{2\pi} \overline{i_{p_n}^2}\, d(\omega t).$$

Values for the plate noise $\overline{i_{p_n}^2}$ of different types of tubes have been computed theoretically[2] and checked closely experimentally (except in the case of diodes) so that the mixer noise can readily be estimated by use of the above averaging process.[3]

It is convenient to express mixer noise in terms of an equivalent noise resistance (except perhaps for the diode mixer). This resistance is defined as that which, if connected across the input of a noise-free mixer with conversion transconductance equal to that of the tube under study, will produce the same noise current in the plate circuit as is present in the actual tube. Thus

$$\overline{e_n^2} = \overline{i_{i-f}^2}/g_c^2 \qquad (17)$$

where g_c is the conversion transconductance and $\overline{e_n^2}$ is the mean-square noise voltage developed by the equivalent noise resistance.

$$R_{eq} = \overline{e_n^2}/(4kT\Delta f) \qquad (18)$$

$$= \overline{i_{i-f}^2}/(4kTg_c^2\Delta f) \qquad (19)$$

where $k = 1.37\times10^{-23}$ joule per degree Kelvin, T is the absolute temperature, and Δf is the noise bandwidth as defined in Part I of this series. As was shown in Part III, it is desirable from a signal-to-noise point of view to have as small a value for R_{eq} as can be obtained.

A convenient table of equivalent noise resistance values for triode and pentode mixers was given by Herold[1] and is reproduced in Table I. In the table, it is assumed that optimum oscillator excitation (i.e., optimized for maximum g_c) is used. The control-grid cutoff voltage is E_{c_0}, the peak grid-to-cathode transconductance (usually taken at zero bias) is g_0, and the peak cathode current (also usually at zero bias) is I_0. (Grid-to-cathode transconductance is defined as the rate of change of cathode current with respect to signal-electrode voltage. It is thus the sum of the transconductances measured between the signal-grid and the screen and plate.) For triodes, $\alpha = 0$, whereas for pentodes, α is the ratio of screen current to plate current. It will be shown in a later section that the average transconductance is of value in estimating the electronic input loading.

In the ultra-high-frequency field, multigrid mixers are not widely used because they are greatly inferior to triodes and pentodes from a signal-to-noise point of view[3] and will not, as a consequence, be discussed here. The two-element mixer such as the diode, on the other hand, has been used to some extent[4] but is sufficiently different in behavior to justify a separate treatment.[5]

TABLE I
Mixer Noise of Triodes and Pentodes
(Oscillator and Signal both Applied to Control Grid)

Operation	Approximate Oscillator Peak Volts	Average Transconductance $\overline{g_m}$	Average Cathode Current $\overline{I_k}$	Conversion Transconductance g_c	Equivalent Noise Resistance R_{eq}
At Oscillator Fundamental	$0.7\,E_{c_0}$	$\frac{0.47}{1+\alpha}\,g_0$	$0.35\,I_0$	$\frac{0.28}{1+\alpha}\,g_0$	$\frac{13}{g_0} + 90\,\frac{I_0}{g_0^2}\,\alpha$
At Oscillator 2nd Harmonic	$1.5\,E_{c_0}$	$\frac{0.25}{1+\alpha}\,g_0$	$0.20\,I_0$	$\frac{0.13}{1+\alpha}\,g_0$	$\frac{31}{g_0} + 220\,\frac{I_0}{g_0^2}\,\alpha$
At Oscillator 3rd Harmonic	$4.3\,E_{c_0}$	$\frac{0.15}{1+\alpha}\,g_0$	$0.11\,I_0$	$\frac{0.09}{1+\alpha}\,g_0$	$\frac{38}{g_0} + 260\,\frac{I_0}{g_0^2}\,\alpha$

IV. Converter or Mixer Gain

The gain in a converter or mixer stage may be treated in exactly the same manner as for the case of the more familiar amplifier stages. The gain is defined as the ratio of the intermediate-frequency voltage on the control grid of the first intermediate-frequency amplifier tube to the signal voltage on the signal electrode of the converter or mixer tube. While the "gain" is a definite quantity which is a measure of the voltage ratio on the grids of successive tubes, it is not a true measure of the "step-up" between stages except at low frequencies where the tube loading is negligible. What is generally of greater interest than gain, is the voltage "step-up" between an antenna and the grid of the first intermediate-frequency tube. At ultra-high frequencies, this is determined not only by the mixer gain, but by the input loading on the signal grid of the mixer. It is thus seen that care must be taken in attempting to arrive at a relative evaluation of different tubes from the standpoint of "gain."

In the general case gain depends upon the conversion transconductance, internal plate resistance of the mixer, and upon the input and transfer impedances of the circuit joining the two tubes. The simple case of a single-tuned circuit joining a pentode mixer to the intermediate-frequency amplifier is illustrated in Fig. 4(a).

[2] B. J. Thompson, D. O. North, and W. A. Harris, "Fluctuations in space-charge-limited currents at moderately high frequency," *RCA Rev.*, vol. 4, pp. 269–285; January, 1940; vol. 4, pp. 441–472; April, 1940; vol. 5, pp. 106–124; July, 1940; vol. 5, pp. 244–260; October, 1940; vol. 5, pp. 371–388; January, 1941; vol. 5, pp. 505–524; April, 1941; vol. 6, pp. 114–124; July, 1941.

[3] E. W. Herold, "Superheterodyne converter considerations in television receivers," *RCA Rev.*, vol. 4, pp. 324–337; January, 1940.

[4] See "40-cm waves for aviation," *Electronics*, vol. 12, pp. 12–15; November, 1939.

[5] See Part V of this series.

The gain may be determined with the aid of the equivalent circuit shown in Fig. 4(b), wherein R_p is the internal plate resistance of the mixer and R is the resonant impedance of the single-tuned network (assumed tuned to the intermediate frequency). $g_c e_s$ represents a constant-current generator, g_c is the conversion transconductance, and e_s is the signal-grid voltage. The voltage developed across R (i.e., the intermediate-frequency voltage across the input to the intermediate-frequency amplifier) is $g_c e_s(RR_p)/(R+R_p)$. This divided by the signal-grid voltage yields the conversion gain $g_c(RR_p)/(R+R_p)$. For pentodes, it is generally the case that $R_p \gg R$, and then conversion gain is equal to $g_c R$. This is similar to the corresponding expression for amplifier gain except that g_c replaces g_m. Under certain conditions the gain of a tube as an ultra-high-frequency amplifier bears a simple relation to its gain when employed as a mixer. In both

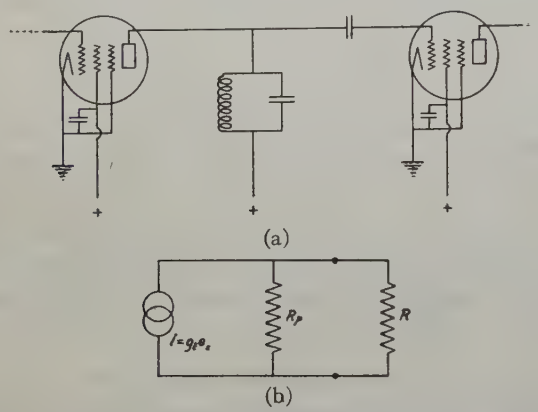

Fig. 4—(a) Circuit shows mixer stage coupled to intermediate-frequency stage by means of single-tuned circuit; (b) circuit which is equivalent of 4(a).

cases it will be assumed that the operating conditions are adjusted for maximum transconductance. It will also be assumed that the tube in each case is operating into a single-tuned circuit of maximum possible resonant impedance. It can be shown that in each case the resonant impedance is given by the expression $Z=1/(2\pi C\Delta f')$ where C is the capacitance of the output circuit and $\Delta f'$ is its effective circuit bandwidth as defined in Part I of this series. Since it is desired that Z be as large as possible, C must be reduced to the unavoidable minimum due to stray tube electrode, lead, and circuit distributed capacitances. The limiting capacitance is largely due to the tube and leads so that the minimum value of C is about the same for either the radio-frequency or intermediate-frequency cases, i.e., for amplifier or mixer operation. Furthermore, at ultra-high frequencies, as well as at intermediate frequencies, the value of $\Delta f'$ is more often determined by the application at hand than by the unavoidable ohmic losses present in the circuit. Under these circumstances, the circuit must be "loaded down" with additional resistance so as to make its effective bandwidth sufficiently great to meet the needs of the application. Since this bandwidth requirement is the same regardless of the carrier frequency employed, $\Delta f'$ is seen to be the same for the amplifier as for the mixer. As a consequence, the maximum output circuit impedance is the same regardless of whether the tube is used as an amplifier or as a mixer. The tube gain for this case has already been shown to be given by gR where g refers to g_m for the amplifier and to g_c for the mixer. Since R, the maximum output circuit impedance, is the same for both cases, the gains will be related in the same fashion as the transconductance. Since, as was indicated above, the maximum conversion transconductance is only about 28 per cent of the maximum amplifier transconductance, it follows that a given tube when used as an amplifier will produce about four times as much gain as when used as a mixer. While this conclusion was drawn for the case of a single-tuned coupling circuit it holds closely for more complicated network cases.

V. Input Loading of Mixer Stage

In Part II of this series, the problem of input loading of vacuum tubes was discussed and expressions for loading due to lead inductances and finite transit angle in the cathode-control-grid region of conventional-type tubes, and the loading in velocity-modulation devices were developed. In mixer operation, the input loading will vary periodically at local-oscillator frequency, so that to determine the actual loading it is necessary to average the instantaneous values over the oscillator cycle. The averaging process is identical with that outlined in Section III where the noise of mixer stages was discussed, and need not be outlined in detail. Since, in many instances, the input conductance varies directly with the transconductance, the average transconductance can be used as a measure of the relative loading.

To see how the mixer loading is related to the instantaneous amplifier loading, let us examine an illustrative case treated by Herold.[1] Fig. 5 depicts the input

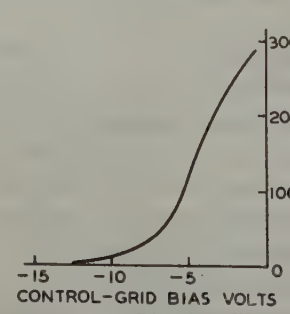

Fig. 5

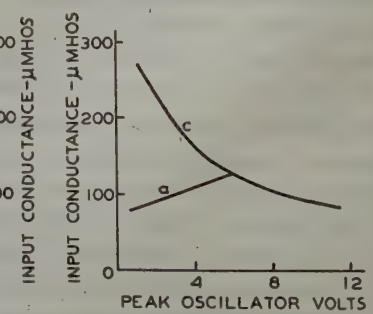

Fig. 6

Fig. 5—Input conductance of a typical variable-μ, radio-frequency pentode at 60 megacycles.

Fig. 6—Input conductance of a typical pentode when used as a mixer at 60 megacycles. *a* fixed-bias operation; *c* bias obtained by means of a high-resistance grid leak.

loading of a "typical pentode" at 60 megacycles as a function of control-grid bias. We consider two cases: 1. the tube is operated at a fixed bias, and 2. the bias is obtained by means of a high-resistance grid leak. The results obtained for mixer operation as a function of local-oscillator voltage (also applied to the screen grid) are shown as curves *a* and *c* of Fig. 6. It is interesting

to note that the loading actually decreases with increasing excitation for the case of grid-leak bias. This is due to the fact that the grid is biased further back towards cutoff with increasing oscillator voltage, so that the cathode current (and consequently, the loading) is cut off over a greater portion of the cycle. Thus, as the oscillator voltage is increased, the loading across the input circuit is reduced, and as consequence R_1 (the resonant impedance of over-all input circuit as defined in Part III) is reduced. For *harmonic* operation the oscillator swing should be further increased for best results, so that R_1 is greater than in the case of optimum fundamental conversion. At the same time, due to decreased g_c, R_{eq} is also increased. It has been shown by Herold[6] that the ratio R_1/R_{eq} in the cases of second- and third-harmonic conversion with triodes is only slightly reduced below the value for fundamental conversion. Since, as was shown in Part III, the signal-to-noise ratio depends largely on R_1/R_{eq}, it follows that for triode converters, the signal-to-noise ratio is not seriously affected by harmonic operation. The gain however, being determined by g_c, is reduced by harmonic operation. In addition, it may be difficult to obtain the larger oscillator excitation required.

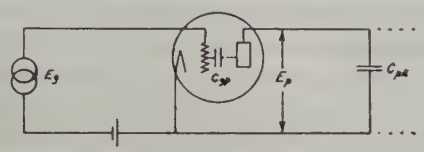

Fig. 7—Effective radio-frequency circuit of a triode mixer.

From the fact that at ultra-high-frequencies the input loading varies as the square of the frequency, the results for any one frequency may be immediately extended to others.

VI. Feedback in Triode Mixers at Ultra-High Frequencies

In general, mixer feedback is due primarily to the capacitance between the plate and signal grid. Since this is appreciable only for triodes, the discussion will be limited to that tube type. A triode mixer feeds into a network designed to have the desired bandwidth at the intermediate frequency. For maximum possible intermediate-frequency circuit impedance (as a condition for maximum gain) the output capacitance should be a minimum. As was previously shown, this is generally accomplished by limiting the output-circuit capacitance to that unavoidably present within the tube and that due to leads and stray and distributed capacitances. Since such inductances as are in the plate circuit are designed so as to result in the desired circuit characteristics at the intermediate frequency, any radio frequency present in the plate circuit will "see" capacitance only. To understand how this output capacitance affects the tube behavior at radio frequency consider the effective triode circuit as shown in Fig. 7. Let the triode be fed by a signal source of negligible internal impedance of voltage amplitude E_g. This will cause a current I_p to flow in the plate circuit of magnitude $\overline{g_m}E_g$, where $\overline{g_m}$ is the average transconductance over the oscillator cycle. This in turn will result in voltage E_p across the effective output capacitance C_{pk} given by

$$E_p = (\overline{g_m}E_g)/(j\omega C_{pk}). \tag{20}$$

A capacitative current will flow in the input circuit, given by

$$I_{gp} = j\omega C_{gp}E_p = \overline{g_m}(C_{gp}/C_{pk})E_g. \tag{21}$$

Therefore the input admittance due to feedback will be

$$A_g = I_{gp}/E_g = \overline{g_m}(C_{gp}/C_{pk}). \tag{22}$$

This is in the form of a conductance which will load down the input circuit, over and above the loading due to ohmic losses, and the loading due to the effects of lead inductance and finite transit angles as was discussed in Part II of this series. Since C_{gp}/C_{pk} may be of the order of 0.3, it is seen that the input resistance due to feedback in a triode mixer may be of the order of several hundred ohms, and may thus have an appreciable effect upon the receiver performance.

VII. Image Frequencies and Intermediate-Frequency Considerations

Let us now consider a problem peculiar to mixers. It is best made clear by an example. If the signal frequency is 100 megacycles and the oscillator frequency is 101 megacycles, the frequencies in the output will include 100 megacycles, 101 megacycles, 201 megacycles (the sum frequency), and 1 megacycle (the difference frequency). Of these frequencies it can be assumed that the desired intermediate frequency is 1 megacycle. The undesired frequencies can be filtered out by the sharply tuned circuits in the intermediate-frequency amplifier. With the oscillator operating at 101 megacycles, it is obvious that an incoming signal of 102 megacycles will provide a difference frequency of 1 megacycle which is also capable of passing through the intermediate-frequency amplifier. Reception of this kind is known as image reception. This image should be avoided in good receiver design.

If a low value is chosen for the intermediate frequency it is difficult to attenuate the image without using tuned-radio-frequency stages ahead of the mixer. This may be undesirable because of practical tuning considerations or from the signal-to-noise standpoint. As a consequence it is generally desirable to use a high value for the intermediate frequency. For ultra-high-frequency applications this may range from 10 to 100 megacycles.

There is another factor which favors the use of high intermediate frequencies. Most ultra-high-frequency applications require the use of a wide-band amplifier following the second detector. Thus, e.g., the final amplification in a television amplifier occurs in the video amplifier which may amplify frequencies up to 4 megacycles. If the intermediate frequency is too low, it

[6] E. W. Herold, "An analysis of the signal-to-noise ratio of ultra-high-frequency receivers," *RCA Rev.*, vol. 6, pp. 302–331; January, 1942.

becomes exceedingly difficult to keep some of it out of the video amplifier with resultant distortions and feedbacks of regenerative or degenerative nature.

VIII. Radio-Frequency Amplification versus Conversion in First Stage of a Receiver

On the basis of the preceding sections of this paper as well as of the earlier parts of this series we are now in a position to render judgment regarding a basic question which arises in the design of all superheterodyne-type receivers. Should one convert immediately in the first stage or should one first use one or more stages of radio-frequency amplification and then convert in a later stage? The answer to this question depends, in general upon the following considerations: 1. Magnitude of received signal; 2. Frequency of received signal; 3. Bandwidth required; 4. Image rejection; 5. Reradiation of local-oscillator power; and 6. Nature of tubes available, i.e., the "state of the art."

The first of these is tied up intimately with the importance of signal-to-noise ratio. If the received signal is large compared with the receiver noise, signal-to-noise ratio is of no importance. This simplifies the consideration of the strong signal case considerably. As was shown above, the gain of an amplifier stage is roughly four times that of a mixer stage. However, since a mixer stage must be used somewhere in a superheterodyne receiver, the lower gain of a mixer must be faced at some point, so that from gain considerations one can draw no positive conclusion concerning the location of the mixer stage. The important factors actually are those related to image rejection and reradiation. If the first stage were a mixer, the image rejection might be inadequate, in which case one or more stages of radio-frequency amplification would be required. By use of a high value of intermediate frequency, immediate conversion may be possible.

In some applications, the radiation from the antenna of local-oscillator power may be a serious factor as regards local interference or in revealing the presence of the receiver. This can be minimized by the use of a radio-frequency stage to isolate the local oscillator from the antenna. If the first stage is a mixer, reradiation can be reduced by the use of a high value of intermediate frequency, so that the antenna and input circuit are far out of tune with the local-oscillator frequency or by the use of balanced (neutralizing) circuits. In general, however, if image rejection and reradiation are not serious, immediate conversion is preferred in order to avoid the difficulties inherent in the tuning of radio-frequency amplifiers. This is particularly the case if the receiver is to be tunable. The bandwidth required and the nature of the tubes employed do not exercise an appreciable role for the case of strong-signal reception and will not be treated further at this point.

If the received signal is weak, the signal-to-noise ratio is of paramount importance. Let us first suppose that for a particular signal frequency and bandwidth application, a tube is available from which a radio-frequency gain considerably greater than unity can be obtained. We postulate further that the bandwidth requirements are such that the circuits must be "loaded down" with external shunt resistance, in order to achieve the desired bandwidth, i.e., in the relation $Z=1/2\pi C\Delta f'$, $\Delta f'$ is determined by the application at hand and not by tube and circuit losses. As has already been pointed out, under these conditions, the mixer gain available from a tube is only about one quarter as great as the available amplifier gain, but since this reduced gain must be faced eventually, no conclusions can as yet be drawn as to whether to make use of radio-frequency amplification or conversion in the first stage. However, the equivalent noise resistance R_{eq} is much lower for the amplifier case than for the mixer, and since this often means a higher signal-to-noise ratio, the use of one or more amplifier stages before conversion is definitely indicated. Now, let us suppose, as a first variation, that the operating frequency is increased. In this case, the circuit impedance which includes the tube loading, decreases with increasing frequency so that the circuit requires less and less additional external loading to achieve the desired bandwidth. Above a certain frequency (which we may call the crossover frequency) the circuit bandwidth, with no external loading, exceeds that required by the application at hand, this being increasingly the case as the frequency goes higher and higher. As a consequence, amplifier gain drops off above the "crossover" frequency. A point will finally be reached for which the amplifier-stage gain drops to unity or lower, in which case it would obviously be foolish to use such a stage, and conversion should be employed in the first stage. In fact, better signal-to-noise response can usually be obtained by immediate conversion for cases in which the amplifier gain is still somewhat above unity. Since the intermediate-frequency-circuit resonant impedance is independent of the signal frequency, the mixer gain remains unaltered as the signal frequency increases, so that the mixer-stage gain may eventually exceed that of the amplifier stage. This constitutes a further argument for immediate conversion above a certain frequency, which we may refer to as the "transition frequency."

The decreased radio-frequency gain with increased frequency is determined largely by tube losses and these in turn depend upon the nature of the tubes available. Thus we may conclude that the transition frequency above which immediate conversion is preferred depends in part, upon the "state of the art." In recent years advances in ultra-high-frequency receiving-tube design have extended the frequency range over which radio-frequency amplification yields better signal-to-noise ratio than is available with converter operation, so that the "transition frequency" is now considerably higher than it was several years ago.

As a second variation, let us suppose that the bandwidth requirements are increased. In that case, the

radio-frequency as well as the intermediate-frequency-circuit impedances must be lowered. As a consequence, since the tube loading plays a lesser role in this case, one can, in general, extend the range of amplifier operation to higher frequencies before the loss in gain becomes so serious as to justify immediate conversion.

IX. Conclusion

If the received signal is strong, one should convert immediately, unless image rejection or the prevention of oscillator radiation necessitate the use of radio-frequency stages. If the received signal is weak, an amplifier stage should be used below a certain frequency and a mixer above, the transition frequency depending upon the characteristics of the tubes available and the bandwidth required. In general the transition frequency occurs at the point where available tubes will no longer give appreciable radio frequency gain for the bandwidth required.

X. Application of Secondary Electron Emission to Superheterodyne Receivers

Secondary emission is ideally suited for application to converters and mixers. It was pointed out in Part II of this series that the gain of a secondary-emission multiplier falls off with increasing frequencies due to the transit-time spreads of secondary electrons. These spreads arise from the fact that different secondaries are emitted with different initial velocities and travel over different paths. This definitely limits the use of secondary-emission multipliers as ultra-high-frequency amplifiers. However, at the intermediate-frequencies used in ultra-high-frequency receivers, the loss in gain described above is negligible so that secondary-emission amplification after conversion is possible and is generally advantageous.

In Part III it was shown that the noise contribution of circuits and tubes following the first stage may be of importance. Anything that can be done to reduce the noise contributed by what follows the first tube will thus improve the signal-to-noise ratio of the complete receiver. The coupling impedance joining the first tube to the second in a conventional-type amplifier is a source of noise and by permitting the plate current of the first tube to go through a secondary-emission multiplier the coupling circuit with its resultant noise contribution is shifted to a later stage where its noise contribution is negligible in comparison with the now greatly amplified noise from earlier sources. One must be sure, however, that the secondary-emission multiplier does not contribute noise in excess of that due to a coupling circuit or resistor. It has been shown[7] that if the secondary-emission ratio n is high, the signal-to-noise ratio is inappreciably affected. If the primary current (plate current for our case) has only pure temperature-limited shot noise ($\overline{i^2}=2eI\Delta f$) then the relative change in the signal-to-noise ratio due to a secondary-emission stage is given approximately by $\sqrt{n/(n+1)}$, which is obviously unimportant if n is large.

[7] V. K. Zworykin, G. A. Morton, and L. Malter, "The secondary emission multiplier—A new electronic device," Proc. I.R.E., vol. 24, pp. 351–376; March, 1936.

Some Aspects of Radio Reception at Ultra-High Frequencies*

E. W. HEROLD†, member, I.R.E. and L. MALTER‡, associate, I.R.E.

Part V. Frequency Mixing in Diodes

E. W. Herold†

Summary—**Although the diode is one of the simplest forms of vacuum tube, the behavior of the diode mixer in superheterodyne reception has not been well understood. One reason for this is that the conversion process is more complex than in other mixers in that it is bilateral, a radio-frequency input voltage giving an intermediate-frequency output current and the resulting intermediate-frequency output voltage in turn giving a radio-frequency current in the input. Analysis of the behavior leads, however, to a very simple equivalent circuit consisting of a symmetrical π circuit of three conductances whose magnitudes are determined by the average diode conductance and by the conversion conductance of the diode. The present paper derives this circuit and uses it to find the conversion loss of the converter stage both with and without input circuit loss. The results, although arrived at independently, are in agreement with the recent publication of James and Houldin.**

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If the conversion loss is to be held small, the diode must be operated so as to obtain the highest ratio of conversion conductance to average conductance. The upper limit of this ratio is unity and this is attained only when the mixer-stage impedance is infinite. Thus, circuit losses prevent the attainment of the condition of no conversion loss, in practice.

The signal-to-noise ratio of a receiver whose input stage is a diode converter is not determinable accurately because of uncertainties in the diode noise behavior. However, by using the conversion loss together with an effective noise temperature for the converter stage, an over-all noise factor can be given in terms of the noise factor of the intermediate-frequency amplifier, F_{i-f} which in the laboratory, is

$$F_{\text{over-all (in laboratory)}}=\frac{F_{i-f}+(1-M)(T_L/T_R-1)}{M}$$

where M is the ratio of intermediate-frequency output power to signal-input power of the converter stage, and T_L/T_R is the ratio of effective noise temperature to room temperature.

The analysis of the average and conversion conductances of particular diodes may be made by use of Fourier analysis in the same manner as with other mixer and converter tubes. The behavior of many diodes is qualitatively shown by an idealized diode whose volt-ampere characteristic is given by two intersecting straight lines. Curves are given showing the conversion loss as a function of the ratio of direct-bias-to-peak oscillator voltage and including input-circuit loss. It is found that conversion at the second or third harmonic of the local oscillator is more critical than at fundamental, but under optimum conditions, the conversion loss is only a few decibels higher.

I. Introduction

THE general theory of superheterodyne frequency conversion is now well understood and it has given a reasonable complete understanding of the behavior as mixers of conventional triodes and multigrid tubes.[1] In the ultra-high-frequency region, however, the use of the two-element nonlinear device (e.g., the diode) as a frequency changer has been common. Although the diode is the simplest type of vacuum tube its behavior as a mixer has been very poorly understood. M. J. O. Strutt[2,3] was early to publish on the subject but failed to make his analysis complete. During intervening years there has been unpublished work, notably that of W. A. Harris of the RCA Victor Division, but it was only a few months ago that a more adequate presentation has appeared in print.[4] The following analysis was developed independently by the present writer some years ago by an extension of the method used for other types of converters or mixers.[1] However, it will be found that the end results are in agreement with those of James and Houldin.[4]

Most mixers and converters using separate signal-input and intermediate-frequency output electrodes are readily analyzed because there is no reverse conversion; i.e., the signal-electrode current is unaffected by the presence or absence of an intermediate-frequency voltage on the output electrode. The diode or two-terminal mixer, however, converts in both directions. The signal-frequency voltage is converted to an intermediate-frequency current and the resulting intermediate-frequency voltage is converted by the local oscillator to a signal-frequency current. Thus, the signal-frequency impedance of the mixer stage is affected by the intermediate-frequency load impedance and the output impedance is affected by the signal circuit. The problem of properly adjusting circuit bandwidths and of matching impedances becomes a complex one.

Finally, the problem of signal-to-noise relations in the two-terminal mixer is very involved. In the first place, the actual fluctuation noise of the two-terminal device must be known under actual operating conditions, i.e., with the local-oscillator voltage applied. Next, the proper connection between signal-frequency and intermediate-frequency noise components must be clearly established since the bidirectional conversion process ties these two components closely together. This aspect of the diode mixer is not yet completely understood.

[1] E. W. Herold, "The operation of frequency converters and mixers," Proc. I.R.E., vol. 30, pp. 84–103; February, 1942.
[2] M. J. O. Strutt, "On conversion detectors," Proc. I.R.E., vol. 22, pp. 981–1008; August, 1934.
[3] M. J. O. Strutt, "Diode frequency changers," *Wireless Eng.*, vol. 13, pp. 73–80; February, 1936.
[4] E. C. James and J. E. Houldin, "Diode frequency changers," *Wireless Eng.*, vol. 20, pp. 15–27; January, 1943.

II. Conversion Theory

1. Basic Analysis

The basic diode-converter stage consists of an input circuit tuned to the signal frequency, an output circuit tuned to the intermediate frequency, and a source of local-oscillator voltage. In operation, a signal-input voltage is impressed on the device and an output voltage of intermediate frequency is also present as a voltage drop across the intermediate-frequency circuit. Since, at the start, the phase relationship of these two voltages is unknown, it is best to proceed by assuming an arbitrary phase relationship, just as if the intermediate-frequency voltage were an *impressed* voltage rather than a voltage drop. After the currents flowing in the circuit have been found, the necessary conditions applicable to the actual case will be obvious and it will be possible to establish the correct phase relationship. The basic circuit is then shown in Fig. 1 where signal-and intermediate-frequency

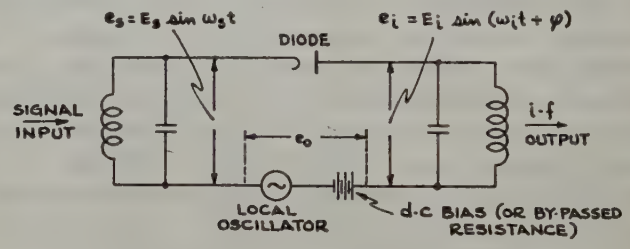

Fig. 1—Schematic circuit of diode converter stage.

voltages are indicated. The intermediate-frequency voltage is given an arbitrary phase angle ϕ.

An analysis will be made which is valid for small signal- and intermediate-frequency voltages, although no restriction is imposed on the magnitude of the local-oscillator voltage. This is in accord with established mixer practice in which comparatively large oscillator voltages are used, but in which the received signals are small. The characteristic of the two-terminal device may be written $i=f(e)$ so that, under mixer conditions,

$$i = f(e_0 + e_i + e_s) \qquad (1)$$

where e_0 is the sum of the local-oscillator voltage and the direct-current bias, e_i is the intermediate-frequency voltage, and e_s is the signal voltage. Since the latter components e_i and e_s are small, a Taylor's expansion may be made about the point of operation determined by e_0,

$$i = f(e_0) + (e_i + e_s)f'(e_0) + \cdots . \qquad (2)$$

Higher-order terms than those shown will be neglected, thus implying that (e_i+e_s) is sufficiently small. The first term of (2) contains only oscillator-frequency terms and is of no interest here. Since the conductance of the two-element device is $g=di/de=f'(e)$ it is evident that the

important part of (2) may be written

$$\begin{aligned} i &= g(e_s + e_i) \\ &= g[E_s \sin \omega_s t + E_i \sin (\omega_i t + \phi)] \end{aligned} \tag{3}$$

where g is the conductance of the two-element device when oscillator voltage and bias only are applied, i.e., when $e = e_0$. Equation (3) might well have been written directly.

It is clear that g, the conductance, varies in time periodically at the frequency of the local-oscillator voltage. The conductance, therefore, may be written as a Fourier series whose fundamental component is at local-oscillator frequency

$$g = g_0 + \sum_{n=1}^{\infty} g_n \cos n\omega_0 t \tag{4}$$

where the cosine series implies that the conductance is single-valued and that the oscillator voltage varies as $\cos \omega_0 t$. The coefficients g_0 and g_n are found by any of the usual methods of harmonic analysis based on the formulas[5]

$$g_0 = \frac{1}{2\pi} \int_0^{2\pi} g \, d(\omega_0 t)$$

$$g_n = \frac{1}{\pi} \int_0^{2\pi} g \cos n\omega_0 t \, d(\omega_0 t).$$

This is exactly the procedure followed in conventional converter theory. Substituting (4) in (3) we get

$$\begin{aligned} i &= g_0 E_s \sin \omega_s t + g_0 E_i \sin (\omega_i t + \phi) + E_s \sum g_n \sin \omega_s t \cos n\omega_0 t) + E_i \sum g_n \sin (\omega_i t + \phi) \cos n\omega_0 t \\ &= g_0 E_s \sin \omega_s t + g_0 E_i \sin (\omega_i t + \phi) + E_s/2 \sum g_n \sin (\omega_s + n\omega_0) t + E_s/2 \sum g_n \sin (\omega_s - n\omega_0) t \\ &+ E_i/2 \sum g_n \sin [(\omega_i + n\omega_0) t + \phi] + E_i/2 \sum g_n \sin [(\omega_i - n\omega_0) t + \phi]. \end{aligned} \tag{5}$$

If any one of the low-frequency components is chosen as the intermediate frequency ω_i, then

$$\omega_i = \pm (\omega_s - n\omega_0) \tag{6}$$

where conversion is at the nth harmonic of the local oscillator. Rearranging (6) it is seen that

$$\pm \omega_i + n\omega_0 = \omega_s. \tag{7}$$

Using (6) and (7) in (5) and disregarding all the terms which contain frequencies other than ω_s or ω_i, the signal-frequency current i_s and the intermediate-frequency current i_{i-f}, are found to be[6]

$$i_s = g_0 E_s \sin \omega_s t + (g_n/2) E_i \sin (\omega_s t + \phi) \tag{8}$$

$$i_{i-f} = g_0 E_i \sin (\omega_i t + \phi) + (g_n/2) E_s \sin \omega_i t. \tag{9}$$

Finally, the original conception of $E_i \sin (\omega_i t + \phi)$ as an *impressed* voltage may be dropped and this term now considered as a *voltage drop*. The new concept may be simplified by assuming the intermediate-frequency load resistance to be tuned to the intermediate-frequency so that it presents a pure resistance R_{i-f} and, at the same time, by assuming the other circuit impedances to be negligible at the intermediate frequency. Then the voltage drop is $E_i \sin (\omega_i t + \phi) = -i_{i-f} R_{i-f}$. Substituting the value of i_{i-f} from (9) and rearranging, it is found that

$$(g_0 + 1/R_{i-f}) E_i \sin (\omega_i t + \phi) = -(g_n/2) E_s \sin \omega_i t. \tag{10}$$

Since this relation must hold for all values of t, it is clear that $\phi = \pi$, and that $E_i/E_s = g_{cn}/(g_0 + 1/R_{i-f})$ where g_{cn} has been written for $g_n/2$. It is seen, of course, that g_{cn} is simply the conversion conductance of the two-element mixer where conversion is at the nth harmonic of the local oscillator. It is analogous in every way to the conversion transconductance of other forms of mixer as given in a previous paper[1] and discussed in Part IV of this series.

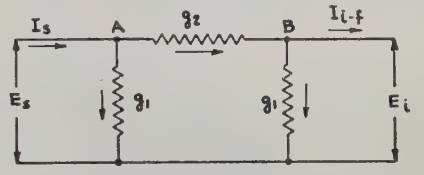

Fig. 2—Symmetrical π network whose equations resemble those for the diode mixer.

Equations (8) and (9) can now be written (putting $\phi = \pi$)

$$i_s = g_0 E_s \sin \omega_s t - g_{cn} E_i \sin \omega_s t \tag{12}$$

$$-i_{i-f} = -g_{cn} E_s \sin \omega_i t + g_0 E_i \sin \omega_i t. \tag{13}$$

It is now possible to solve (10), (12), and (13) and find all the desired quantities. However, it is not necessary to do this formally as will now be shown.

2. Equivalent Circuit for the Diode-Converter Stage

There is a similarity between (12) and (13) of the foregoing and the mesh equations for a symmetrical network of three conductances arranged in a π. Considering the symmetrical π network of Fig. 2, the Kirchhoff-law relation may be written for the sum of currents at points A and B respectively. It is seen that

$$\begin{aligned} I_s &= E_s g_1 + (E_s - E_i) g_2 \\ -I_{i-f} &= E_i g_1 - (E_s - E_i) g_2 \end{aligned}$$

where currents entering the points are equated to those leaving the points. Rearranging it is found

$$I_s = E_s(g_1 + g_2) - g_2 E_i \tag{14}$$

$$-I_{i-f} = -E_s g_2 + (g_1 + g_2) E_i. \tag{15}$$

A comparison of (14) and (15) with (12) and (13) shows that the mixer circuit may be considered as equivalent to Fig. 2 as far as amplitudes of currents and voltages are concerned. As to frequencies, of course, the mixer circuit is always different from the passive network inasmuch as the frequency of the output voltage and current differs from the input frequency.[7] Furthermore,

[5] For derivation see any standard text, e.g., E. B. Wilson's, "Advanced Calculus," Ginn and Company, New York, N. Y., p. 458.

[6] It may be noted that the magnitude of the local-oscillator voltage E_0 does not appear directly in the current relations. However, the Fourier coefficients g_0 and g_n depend on the local-oscillator voltage and thus the currents are dependent on E_0 to some extent.

[7] When the intermediate-frequency circuit has appreciable impedance at radio frequency or when the radio frequency circuit has impedance at intermediate frequency, these impedances should be included in the radio-frequency and intermediate-frequency branches of the circuit, respectively.

if $g_0=g_1+g_2$ and $g_2=g_{cn}$ so that $g_1=g_0-g_{cn}$ all the solutions to the mixer problem are found by simply solving the circuit of Fig. 3. Almost every relation of importance can be written down by inspection of this simple equivalent circuit.[8]

III. Impedances and Conversion Loss of Mixer Stage

1. *Maximum Power Transfer: Image or Iterative Impedance*

It can be shown[9] that, barring negative resistance effects, the conversion conductance g_{cn} can never exceed the average conductance g_0. Thus, in examining Fig. 3, the shunt arms may be considered as positive conductances and it is clear that the intermediate-frequency output voltage can never exceed the radio-frequency signal voltage, no matter how high the

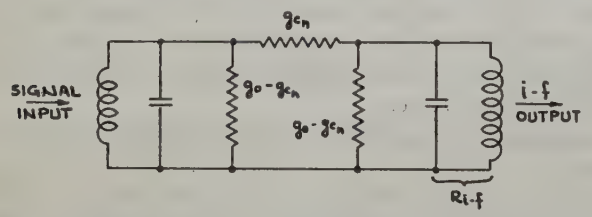

Fig. 3—The equivalent circuit for the diode mixer. This circuit gives the correct *magnitude* for input and output currents and voltages; however, the frequency of the input current and voltage is different from the output frequency.

intermediate-frequency circuit impedance is. The further discussion of the evaluation of g_0 and g_{cn} will be left for a later section. For the present, let the characteristics of the diode mixer stage be examined as they are indicated by the equivalent passive network of Fig. 3.

To begin with, it is clear that the mixer stage is basically a symmetrical π-type attenuator which, at the same time changes the frequency. If it is connected to the signal source and to the intermediate-frequency load through ideal transformers, maximum power transfer will occur when the transformers are adjusted to match the iterative or image impedance. This impedance is that which, when used as a termination, makes the input impedance of the mixer stage equal to this terminating value. In other words, by use of this impedance as a termination at each end, impedance matching is maintained throughout. The input conductance, when the intermediate-frequency conductance is g_x, can be written by inspection of Fig. 3.

$$g_{in} = g_0 - g_{cn} + \frac{g_{cn}(g_0 - g_{cn} + g_x)}{g_0 + g_x} = \frac{g_0^2 - g_{cn}^2 + g_0 g_x}{g_0 + g_x}. \quad (16)$$

If this input conductance is equated to g_x then g_x becomes what might be called the iterative conductance and is given by

$$g_x^2 = g_0^2 - g_{cn}^2. \quad (17)$$

[8] To the writer's knowledge this equivalent circuit was first used by W. A. Harris of the RCA Victor Division.

[9] See Section IV, 1, of this paper.

This is one of the more important characteristics of the mixer stage.

2. *Conversion Loss Under Matched Conditions: Circuit Losses Neglected*

The minimum power loss in changing the signal frequency to the intermediate frequency occurs when the input and output are matched to g_x by ideal, loss-free circuits; the output-to-input power ratio is then given by the square of the voltage ratio (since the impedances are the same). Again examining Fig. 3 it is seen that if a voltage E_s is impressed across the signal-input terminals, the voltage across the intermediate-frequency output E_i is by inspection

$$E_i = E_s \frac{(g_0 - g_{cn} + (1/R_{i-f}))g_{cn}}{g_0 + 1/R_{i-f}} \cdot \frac{1}{g_0 - g_{cn} + 1/R_{i-f}} = E_s \frac{g_{cn}}{g_0 + 1/R_{i-f}}. \quad (18)$$

If the intermediate-frequency load is matched to the diode mixer, then $1/R_{i-f}=g_x$ and, using (17),

$$\frac{E_i}{E_s} = \frac{g_{cn}}{g_0 + \sqrt{g_0^2 - g_{cn}^2}} = \frac{g_{cn}/g_0}{1 + \sqrt{1 - (g_{cn}/g_0)^2}} \quad (19)$$

Since the input and output impedances are the same, this relation may be expressed as a conversion loss in decibels; a curve of conversion loss is plotted against g_{cn}/g_0 in Fig. 4.

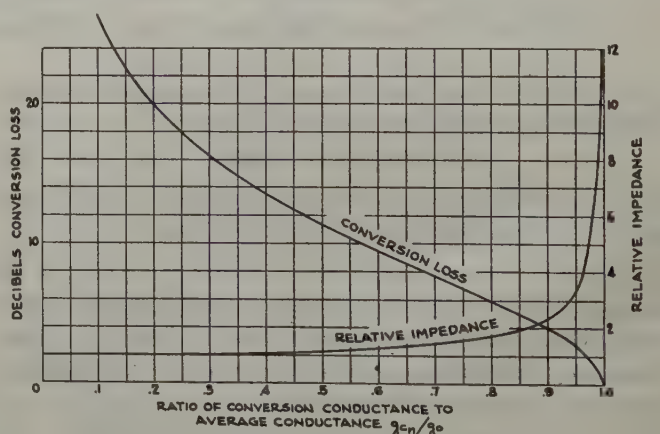

Fig. 4—The behavior of a diode mixer as a function of the two basic conductance components, assuming no loss in the input or output transformers and impedance matching throughout.

It is seen from Fig. 4 that, unless g_{cn}/g_0 is approximately unity the conversion loss is appreciable. However, an important factor to consider is the absolute magnitude of the matching conductance g_x. If this is very small (i.e., a very high impedance) it is not possible to match properly without the matching-circuit losses playing an important role. As g_{cn}/g_0 approaches unity, g_x approaches zero and the impedances needed to match approach infinity. The relative matching impedance g_0/g_x is plotted on a second curve on Fig. 4.

As will be seen later, the only way in which g_{cn}/g_0

can approach unity in a diode-mixer stage is for g_{cn} and g_0 separately to become very small. This in itself implies a very high matching impedance. If a particular diode is chosen, and by adjustment of operating conditions g_{cn}/g_0 is made to approach unity, the curve of matching impedance would rise much more steeply than that shown in Fig. 4.

It may be concluded first, that the diode mixer is not operable as a power-transfer device without appreciable conversion loss, and second, that circuit losses must be considered for any diode mixer whose conversion loss is small. Thus, it is logical to consider the next topic.

3. Conversion Loss Including Losses in Input Circuit

If the input circuit has a shunt loss which may be represented by a conductance g_s, the circuit becomes similar to the one shown in Fig. 5(a). For the purposes

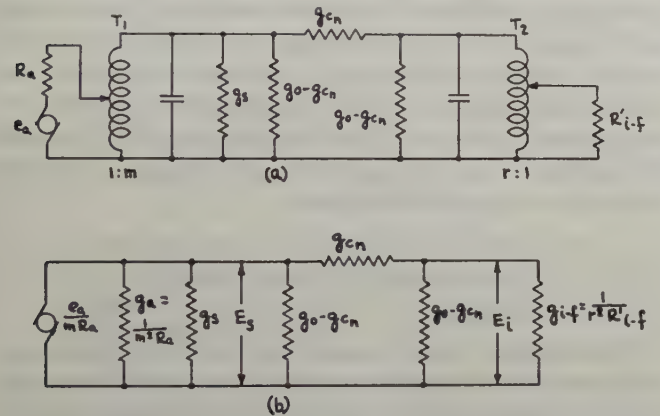

Fig. 5—(a) The schematic circuit of a diode mixer including input-circuit loss, shown as a conductance g_s. (b) An equivalent circuit wherein reflected current and conductances are substituted for the input and output transformers.

of this analysis, T_1 and T_2 may be assumed to be ideal transformers whose step-up adjustments permit optimum power transfer to be obtained from the signal source e_a to the intermediate-frequency load R'_{i-f}. The transformers T_1 and T_2 may be removed to give the equivalent circuit of Fig. 5(b) by inserting the impedances as seen from the diode-mixer input and output terminals. The conversion loss can now be computed from the ratio of power dissipated in the intermediate-frequency conductance, g_{i-f}, to that dissipated in the input, i.e., g_s combined with the diode input conductance. The power dissipated in g_{i-f} is output power $= E_i^2 g_{i-f}$ where E_i is the voltage across the output terminals (see Fig. 5). The input power is input power $= E_s^2(g_s + g_{in})$ where E_s is the voltage across the input terminals and g_{in} is the diode input conductance. From (16)

$$g_{in} = \frac{g_0^2 - g_{cn}^2 + g_0 g_{i-f}}{g_0 + g_{i-f}}$$

and from (18)

$$\frac{E_i}{E_s} = \frac{g_{cn}}{g_0 + g_{i-f}}.$$

Using these relations

$$\frac{\text{output power}}{\text{input power}} = \left(\frac{E_i}{E_s}\right)^2 \frac{g_{i-f}}{g_s + g_{in}} = \left(\frac{g_{cn}}{g_0 + g_{i-f}}\right)^2 \frac{g_{i-f}}{g_s + \dfrac{g_0^2 - g_{cn}^2 + g_0 g_{i-f}}{g_0 + g_{i-f}}}.$$

Multiplying out, it will be found that this becomes

$$\frac{\text{output power}}{\text{input power}} = \frac{g_{cn}^2}{(2g_0^2 + 2g_0 g_s - g_{cn}^2) + (g_0/g_{i-f})(g_0^2 - g_{cn}^2 + g_s g_0) + g_{i-f}(g_0 + g_s)}. \tag{20}$$

Considering g_{i-f} as a variable, the denominator contains the form $(ax^{-1} + bx)$ which was treated in Section II, 2, of Part III of this series. The maximum power transfer (minimum denominator) occurs when

$$g_{i-f}^2 = \frac{g_0(g_0^2 - g_{cn}^2 + g_s g_0)}{g_0 + g_s} \tag{21}$$

and is

$$M = \left[\frac{\text{output power}}{\text{input power}}\right]_{max} = \frac{g_{cn}^2}{2g_0^2 + 2g_0 g_s - g_{cn}^2 + 2\sqrt{g_0(g_0 + g_s)(g_0^2 - g_{cn}^2 + g_s g_0)}} = \frac{g_{cn}^2}{\left[\sqrt{g_s(g_0 + g_s)} + \sqrt{g_0^2 - g_{cn}^2 + g_s g_0}\right]^2} = \left[\frac{g_{cn}/g_0}{\sqrt{1 + g_s/g_0} + \sqrt{1 - (g_{cn}/g_0)^2 + g_s/g_0}}\right]^2. \tag{22}$$

Since g_{cn}/g_0 can approach unity only as g_{cn} and g_0 each approach zero, the effect of the shunt-input loss g_s is to impose a practical optimum value for g_{cn}/g_0 which is less than unity. Thus, the conclusion as to the impossibility of attaining conversion without appreciable loss seems to be further strengthened. In Figs. 9, 10, and 11 of a later section, curves will be shown for an idealized diode-mixer stage which show how g_{cn} and g_0 will vary with the operating parameters and which also illustrate the minimum conversion loss imposed by input-circuit loss.

4. The Over-All Signal-to-Noise Ratio

The over-all signal-to-noise ratio of a receiver with a diode-converter stage is a quantity of considerable importance but is not accurately predictable on theoretical grounds. There are several reasons for this statement. In the first place, the measured fluctuation noise in space-charge-limited diodes is usually considerably larger than would be expected by theory. The discrepancy was satisfactorily explained by North[10] as due to elastically reflected electrons which disturbed the virtual cathode so as to upset the space-charge reduction of shot noise. Thus, an accurate measure of the noise is not available in general, and presumably changes in anode structure or material of a diode may alter the noise without in other ways affecting the diode performance. Second,

[10] D. O. North, Part II of "Fluctuations in space-charge-limited currents at moderately high frequencies," *RCA Rev.*, vol. 5, pp. 117–124; July, 1940.

diode-noise fluctuations affect both the radio-frequency and the intermediate-frequency circuits and are not necessarily independent of each other. Finally, large-signal, transit-time effects cannot always be neglected at the ultra-high frequencies and may have a bearing on the noise behavior; such effects have not yet been fully investigated.

A rough qualitative notion of the effect of the diode-mixer stage on the signal-to-noise ratio of a receiver can be made, however. For one thing, the conversion loss cuts the signal available at the input of the intermediate-frequency amplifier so that the signal-to-noise ratio will be worse than that of the intermediate-frequency amplifier by a predictable amount on this score alone. If the diode-mixer stage adds some noise, and it usually will, this decreases the over-all signal-to-noise ratio still more.

It can be seen that the converter stage acts as an attenuator of the signal and as an additional noise source itself. Thus, a radio receiver may consist of an antenna of radiation resistance R_a, whose effective noise temperature is T_a, and which is connected to the intermediate-

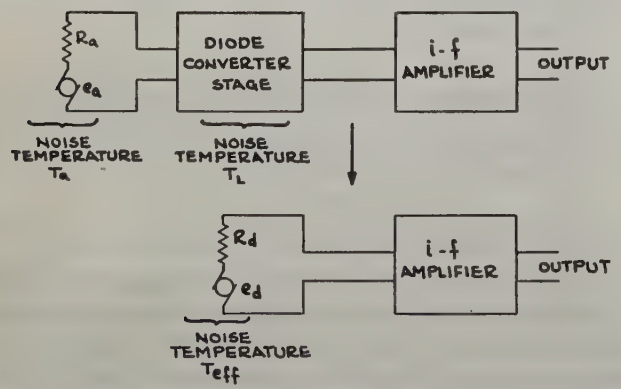

Fig. 6—A receiver consisting of an antenna, a diode mixer, and an intermediate-frequency amplifier may be analyzed for signal-to-noise ratio by replacing the antenna and diode mixer by an equivalent resistor at a noise temperature T_{eff}.

frequency receiver through the mixer as in Fig. 6. We may substitute for this combination of antenna and mixer, a resistance at the input to the intermediate-frequency amplifier by placing this resistor at an effective temperature T_{eff} for noise purposes and by assigning an available signal power (e_d^2/R_d of Fig. 6) which is at a lower frequency and is less than that of the actual antenna by the conversion loss of the mixer. This is almost exactly the problem which was treated in Section II, 7, of Part III of this series where the effect on signal-to-noise ratio of a passive transducer between antenna and receiver was worked out. Thus, if the diode-mixer stage is considered as a noise source at an effective noise temperature T_L, reference to Part III gives

$$T_{eff} = T_L(1 - M) + MT_a \qquad (23)$$

where M is the conversion loss expressed as the ratio of output power to input power of the diode mixer (equation (22) above).

The intermediate-frequency system may be measured as if it were a receiver by itself, using a dummy antenna of value R_d (the output impedance of the diode mixer) as shown in Fig. 6. If this dummy antenna is at room temperature, as in the laboratory, the intermediate-frequency system will have a noise factor[11] F_{i-f}. The over-all receiver including the diode-mixer stage will then have a noise factor

$$F_{\text{over-all}} = \frac{F_{i-f} - 1 + T_{eff}/T_a}{M}. \qquad (24)$$

In the laboratory, an over-all measurement would be made with a dummy antenna so that $T_a = T_R$ giving

$$T_{eff}/T_R = M + (1 - M)T_L/T_R$$

so that

$$F_{\text{over-all (in laboratory)}} = \frac{F_{i-f} + (1-M)(T_L/T_R - 1)}{M}. \qquad (25)$$

These relations are only of indirect value unless the effective noise temperature T_L of the converter stage is known. The measurement of F_{i-f}, $F_{\text{over-all}}$ and the conversion loss, M, will permit T_L/T_R to be calculated from (25).

The effective temperature will not be independent of operating conditions of the diode and may be substantially higher than would be expected on the basis of the highest temperature element in the converter stage. For a diode mixer with an oxide-coated cathode at 1000 degrees Kelvin, it would be improbable that T_L/T_R could be less than 2 and not out of reason to expect values of this ratio as high as 10 or more.

IV. The Evaluation of Average and Conversion Conductances

1. The General Case

It has already been brought out that best mixer performance will be obtained when the conversion conductance g_{cn} approaches the average conductance g_0, provided circuit losses can be neglected. The instantaneous conductance has been written as a Fourier series (equation (4)) so that the quantities g_0 and g_{cn} are given directly in terms of the Fourier coefficients,

$$g_0 = \frac{1}{2\pi}\int_0^{2\pi} g\, d(\omega t) \qquad (26)$$

$$g_{cn} = \frac{g_n}{2} = \frac{1}{2\pi}\int_0^{2\pi} g \cos n\omega t\, d(\omega t). \qquad (27)$$

If negative values of g are excluded, inspection indicates that, since the integrand of the second expression can never exceed that of the first (since cos $n\omega t$ can never exceed unity), the integral in the second case can never exceed the integral in the first case, i.e., the conversion conductance g_{cn} can never exceed the average conductance g_0. However, if the conductance g is an impulsive-type function of finite maximum value and if, during the entire time at which g is greater than zero, cos $n\omega t$ is substantially unity, then the two integrals will each become small and will approach equality. It is therefore seen that, only in the relatively ineffectual case where both g_0 and g_{cn} are very small, can the two approach equality.

[11] See discussion of noise factor in Part III of this series.

It should also be noted that, in general, the Fourier series for g (equation (4)) will be convergent, so that conversion at oscillator harmonics is progressively less efficient as the order of harmonic is raised. Again, with the ineffectual impulsive function, all the conversion conductances (at different oscillator harmonics) approach each other in magnitude, but only as they all approach zero.

For a diode, or any other two-element nonlinear device for that matter, the mixer behavior can be estimated from its conductance-versus-voltage characteristic. From this characteristic, the conductance-versus-time curve may be obtained for any particular local-oscillator injection voltage, and a Fourier analysis made. It is more usual to make an analysis directly from points on the curve of conductance versus bias, and some simple 7-point formulas for g_{c1}, g_{c2}, and g_{c3} have been given.[1]

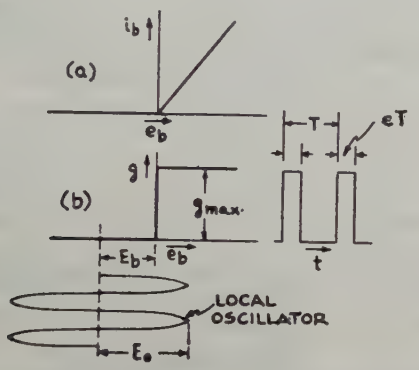

Fig. 7—(a) Plate-current-versus-plate-voltage characteristic of an idealized diode. (b) Conductance characteristic of idealized diode showing an applied oscillator with a bias E_b and the resulting rectangular pulses of conductance versus time.

It is also necessary to obtain g_0, in the present instance. Using Fig. 2 (a) of reference 1 it may be shown that, to a sufficient accuracy for most purposes,

$$g_0 = 1/12[g_1+g_7+2(g_2+g_3+g_4+g_5+g_6)] \qquad (28)$$

where the g_1, g_2, etc., are points on the curve of the figure.[12] The formula for g_{c1} as given in the reference is

$$g_{c1} = 1/12[(g_7-g_1)+(g_5-g_3)+1.73(g_6-g_2)]. \qquad (29)$$

It is clear by comparing these formulas for g_0 and g_{c1} that highest g_{c1} and lowest g_0 can be obtained by operating the device so that $g_1=g_2=g_3=0$, if this is possible. It is interesting to note that no use is ordinarily found for the diode current-versus-voltage characteristic, except when the average direct current is required.

2. *An Idealized Diode*

Much can be learned from an examination of the behavior of the idealized diode whose current characteristic is shown in Fig. 7 (a) and whose conductance characteristic is shown in Fig. 7 (b). If a local-oscillator voltage, of peak value E_0, is applied, together with a direct-current bias of value E_b (from a battery or a by-passed resistor) then the conductance versus time will be the rectangular pulse function shown to the right of Fig. 7 (b). Such a function has Fourier components such that

$$g_0 = g_{max}\epsilon \qquad (30)$$

$$g_{cn} = g_{max}/\sin n\pi\epsilon/n\pi \qquad (31)$$

where ϵ is the fraction of the time during which current flows, i.e.,

$$\pi\epsilon = \cos^{-1}(E_b/E_0). \qquad (32)$$

Thus, when ϵ is small, it is seen that g_0 and g_{cn} are approximately equal and the conversion conductances at the different harmonics are all nearly equal but are all very small. Fig. 8 shows curves of the average diode conductance and of g_{c1}, g_{c2}, and g_{c3} as the ratio of bias voltage to peak-oscillator voltage is varied. It is seen that harmonic operation requires a greater bias than fundamental operation.

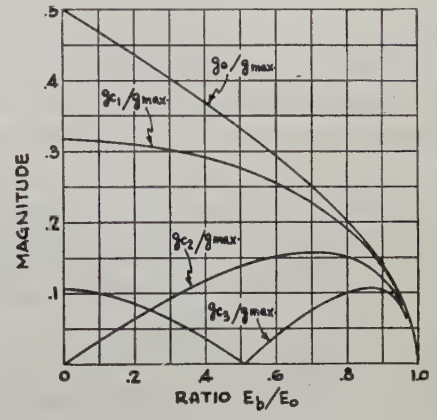

Fig. 8—Average conductance g_0 and conversion conductances g_{c1}, g_{c2}, and g_{c3} for the idealized diode of Fig. 7 as a function of the ratio of direct-current bias to peak oscillator voltage. The curves are plotted as ratios of the conductance to the maximum diode conductance g_{max}.

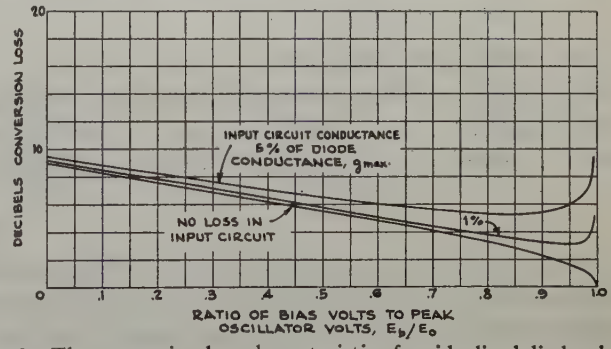

Fig. 9—The conversion-loss characteristic of an idealized diode whose maximum conductance is g_{max}. The curves are for conversion at oscillator fundamental and show the effect of loss conductance in the input circuit.

Of more direct interest are the curves of Fig. 9 which show the conversion loss, using oscillator fundamental, for the idealized diode, again as a function of the ratio of bias to peak oscillator voltage. These curves were computed from (22) and therefore represent optimum values. The curve for a perfect no-loss input circuit is an impracticable ideal which is shown only for comparison. The other curves give the results for an input circuit whose conductance g_s is 1 and 5 per cent, respectively, of the diode maximum conductance g_{max}.

[12] They are not to be confused with the g's of the Fourier series used in the present paper in equation (4).

A similar set of curves for conversion at second and third harmonics are shown in Figs. 10 and 11. It is interesting to compare the optimum results for conversion at a harmonic with normal conversion at fundamental. Taking the minimum conversion losses from Figs. 9, 10, and 11, Table I may be prepared. It is seen that harmonic operation is by no means out of the question if a decibel or two more conversion loss can be tolerated. However, Figs. 9, 10, and 11 show that the adjustment for lowest conversion loss is much more critical when harmonic conversion is used.

TABLE I

MINIMUM CONVERSION LOSS OF IDEALIZED DIODE

Operation at	Decibel Loss $g_s = 0.01\ g_{max}$	Decibel Loss $g_s = 0.05\ g_{max}$
Fundamental	3.2	5.3
Second Harmonic	4.0	6.6
Third Harmonic	4.6	7.6

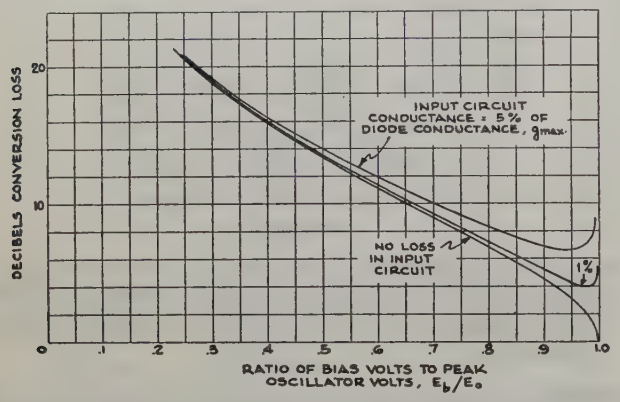
DECIBELS CONVERSION LOSS
20
10
0
INPUT CIRCUIT CONDUCTANCE = 5% OF DIODE CONDUCTANCE, g_{max}
NO LOSS IN INPUT CIRCUIT
1%
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
RATIO OF BIAS VOLTS TO PEAK OSCILLATOR VOLTS, E_b/E_o

Fig. 10—Conversion loss of an idealized diode with second-harmonic conversion. The curves show the effect of loss conductance in the input circuit.

3. *Discussion of Practical Diodes*

Although the idealized diode which has been discussed in Section IV, 2, is not realizable in practice, the behavior shown is qualitatively applicable to practical diodes. Particular characteristic-curve shapes may be more accurately analyzed by use of the general formulas in Section IV, 1, together with (22). The application of the integral formulas (26) and (27) to such a common characteristic as the 3/2-power law leads to results in terms of elliptic functions[4] and it is often more rapid to use the 7-point approximate formulas (28) and (29). This is certainly true for more complex characteristics for which the analytic expression is difficult to handle.

When, as in practical diodes, the conductance rises gradually with an increase in applied voltage up to the value g_{max}, the results will, as a rule, be inferior to those shown in Figs. 9, 10, and 11. However, a large oscillator swing together with appropriately large bias does permit an approach to the results shown. In some instances, it may be necessary to take into account conduction in both directions. It is clear that this phenomenon increases the average conductance and decreases the conversion conductance under any conditions whatever, so that it becomes impossible to equal the performance of the idealized diode. Analysis shows that there is then an optimum operating condition even if the input circuit is loss-free, and this optimum, of course, has a finite conversion loss.

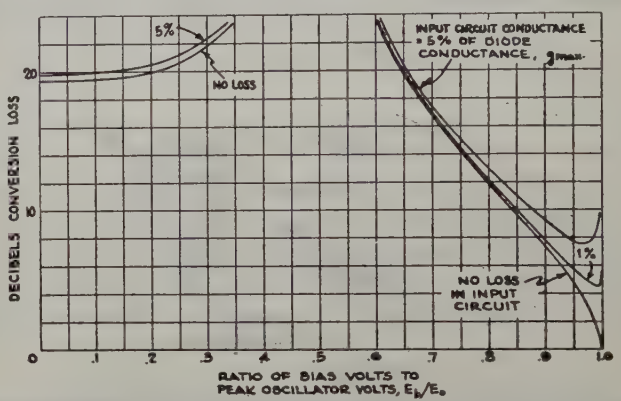

Fig. 11.—Conversion loss of an idealized diode with third-harmonic conversion. The effect of loss in the input circuit is shown.

Correction

It has been brought to the attention of the author, Frederick Emmons Terman, that an error appears in his book *Radio Engineers Handbook*, which has been recently published. This error is in equation (117) which appears on page 216 of the *Handbook*. This particular section of the *Handbook* was reprinted in the April, May, and June, 1943, issues of the PROCEEDINGS under the title of "Network Theory, Filters, and Equalizers." The error in question appears on page 234 of the May, 1943, issue of the PROCEEDINGS in equation (38). In place of reading

$$R_1 = R_1\left(\frac{\alpha - 1}{\alpha + 1}\right)$$

$$R_2 = R_1\left(\frac{2\alpha}{\alpha^2 + 1}\right)$$

it should be

$$R_1 = R_1\left(\frac{\alpha - 1}{\alpha + 1}\right)$$

$$R_2 = \left(\frac{2\alpha}{\alpha^2 - 1}\right).$$

THE INSTITUTE OF RADIO ENGINEERS

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DALLAS-FORT WORTH—Chairman, H. E. Applegate; Secretary, P. C. Barnes, WFAA-WBAP, Grapevine, Texas.

DETROIT—Chairman, F. M. Hartz; Secretary, E. J. Hughes, 14209 Prevost, Detroit, Mich.

EMPORIUM—Chairman, R. K. Gessford; Secretary, H. D. Johnson, Sylvania Electric Products, Inc., Emporium, Pa.

INDIANAPOLIS—Chairman, A. N. Curtiss; Secretary, E. E. Alden, WIRE, Indianapolis, Ind.

KANSAS CITY—Chairman, B. R. Gaines; Secretary, R. N. White, 4800 Jefferson St., Kansas City, Mo.

LOS ANGELES—Chairman, Lester Bowman; Secretary, R. C. Moody, 4319 Bellingham Ave., North Hollywood, Calif

MONTREAL—Chairman, L. T. Bird; Secretary, J. C. R. Punchard, Northern Electric Co., 1261 Shearer St., Montreal, Que., Canada.

NEW YORK—Chairman, H. M. Lewis; Secretary, H. F. Dart, 33 Burnett St., Glen Ridge, N. J.

PHILADELPHIA—Chairman, W. P. West; Secretary, H. J. Schrader, Bldg. 8, Fl. 10, RCA Manufacturing Co., Camden, N. J.

PITTSBURGH—Chairman, B. R. Teare; Secretary, R. K. Crooks, Box, 2038, Pittsburgh, Pa.

PORTLAND—Chairman, K. G. Clark; Secretary, E. D. Scott, Rt. 14, Box 414, Portland, Ore.

ROCHESTER—Chairman, O. L. Angevine, Jr.; Secretary, G. R. Town, Stromberg-Carlson Tel. Mfg. Co., Rochester, N. Y.

ST. LOUIS—Chairman, N. J. Zehr; Secretary, H. D. Seielstad, 1017 S. Berry Rd., Oakland, St. Louis, Mo.

SAN FRANCISCO—Chairman, Karl Spangenberg; Secretary, David Packard, Hewlett-Packard Co., Palto Alto, Calif.

SEATTLE—Chairman, L. B. Cochran; Secretary, H. E. Renfro, 4311 Thackeray Pl., Seattle, Wash.

TORONTO—Chairman, T. S. Farley; Secretary, J. T. Pfeiffer, Erie Resistor of Canada, Ltd., Terminal Warehouse Bldg., Toronto, Ont., Canada.

TWIN CITIES—Chairman, E. S. Heiser; Secretary, B. R. Hilker, KSTP, St. Paul Hotel, St. Paul, Minn.

WASHINGTON—Chairman, C. M. Hunt; Secretary, H. A. Burroughs, Rm. 7207, Federal Communications Commission, Washington, D. C.

Institute News and Radio Notes

Board of Directors

A special meeting of the Board of Directors took place on July 21, 1943, and was attended by L. P. Wheeler, president; F. S. Barton, vice president; S. L. Bailey, W. L. Barrow, I. S. Coggeshall, W. L. Everitt, H. T. Friis, Alfred N. Goldsmith, editor; O. B. Hanson, R. A. Heising, treasurer; F. B. Llewellyn, Haraden Pratt, secretary; B. J. Thompson, H. M. Turner, H. A. Wheeler, W. C. White, and W. B. Cowilich, assistant secretary.

Secretary Pratt, in his capacity as chairman of the Radio Technical Planning Association, reported on the further progress made in the development of that agency and submitted the tentative draft of the "Organization and Procedure" which had been prepared jointly by the Institute and Radio Manufacturers Association committees. It was indicated that another meeting of the two committees would take place in August for the purpose of completing the plan.

Among the actions taken relative to the RTPA, which was discussed at length, were the appointments of Secretary Pratt to serve as representative of the Institute, and Mr. Thompson as alternate, when the named agency is established.

The Consultative Committee on Engineering (advisory to the War Manpower Commission) was discussed in connection with the report on that body, given by Secretary Pratt. It was noted that a number of other engineering societies had accepted the proposal to have all engineering-manpower matters, to be directed to the attention of the Government, clear through the Consultative Committee on Engineering. The proposal was favorably regarded but final endorsement was reserved for subsequent action to be taken when the complete plan is made available.

President Wheeler announced that the Executive Committee approved the preparation and submission of a supplementary appeal to the War Production Board for relief from the modified Paper Limitation Order L-244 of that agency, which places severe restrictions on the supply of paper made available for printing the PROCEEDINGS.

On recommendations of the Executive Committee, increased budgets were approved including those for the printing of the PROCEEDINGS and the Cumulative Index (1913–1942).

The 1942 Report of the Secretary was unanimously accepted and approved.

Mr. Coggeshall, chairman of the Public Relations Committee, reported that letters on and copies of the Institute's resolution on Senate Bill S-702 had been extensively distributed to the technical and lay press. Attention was also called to the replies concerning the resolution, received from Senator H. M. Kilgore and Representative Wright Patman.

Dr. Terman was appointed to serve as the representative of the Institute on the Division of Engineering and Research of the National Research Council, Washington, for a term of three years beginning July 1, 1943.

A temporary committee on Institute's investments consisting of President Wheeler, Treasurer Heising, and Secretary Pratt was appointed to replace the present special committee.

The following appointments were also made: F. A. Polkinghorn, Admissions Committee; J. J. Callahan, Facsimile Committee; and H. N. Blackmon, R. K. Honaman, and E. L. Robinson, Papers Procurement Committee.

Unanimous approval was granted to these applications: for transfer to Member grade from R. G. Clark, E. N. Dingley, Jr., and A. J. Ebel; for admission to Member grade from L. J. Purgett; 19 for transfer to Associate grade; and 137 for Associate, 109 for Student, and 4 for Junior grades.

The area of the Chicago Section was enlarged to include the addition of the following counties, as recommended by the Executive Committee: Illinois—Champaign, Ford, Iroquois, Kankakee, Livingston, Marshall, McLean, Peoria, Putnam, Tazewell, Vermillion, Winnebago, and Woodford; Indiana—St. Joseph; Wisconsin—Dane, Jefferson, Kenosha, Milwaukee, Racine, Rock, Walworth, and Waukesha.

The printing of the Temporary Facsimile Test Standards, recently completed by the Facsimile Committee, and other temporary Standards reports during the period of the present war, was approved.

R. H. Dishington was conditionally appointed Institute Representative at the University of Southern California.

Executive Committee

The Executive Committee met on July 20, 1943, and those present were L. P. Wheeler, chairman; Alfred N. Goldsmith, editor; R. A. Heising, treasurer; F. B. Llewellyn, Haraden Pratt, secretary; H. A. Wheeler, and W. B. Cowilich, assistant secretary.

The applications for transfer to Member grade in the names of R. G. Clark, E. N. Dingley, Jr., and A. J. Ebel, and for admission to Member grade in the name of L. J. Purgett, were recommended to the Board of Directors for approval.

The 137 applications for admission to Associate, 109 to Student, and 4 to Junior grades, and the 19 applications for transfer to Associate grade, were also approved for confirming action by the Board of Directors.

Assistant Secretary Cowilich reported on office-personnel matters including the overtime work during the month of May, the setting up of the required bookkeeping records for the "Pay-As-You-Go" income tax, and the progress of the preparation of the Audit Bureau of Circulations statement for the first six-month period of 1943.

Chairman Wheeler stated that on July 1, 1943, he and Editor Goldsmith met with a representative of the War Production Board, at Washington, relative to seeking relief for the PROCEEDINGS from that agency's modified Paper Limitation Order L-244. It was learned that WPB does not intend to make available to the Institute the paper tonnage that would be required to print the PROCEEDINGS at the present rate of increase in membership and in number of pages per issue.

Under the circumstances Editor Goldsmith announced that, effective with the August issue, the covers and inside pages of the PROCEEDINGS would of necessity be printed on still lighter-weight paper, certain typographical and related space-saving changes would be accomplished, and the number of surplus copies and pages per issue printed would be reduced.

In addition to making the reasonable paper economies, it was agreed to prepare and submit as soon as possible another supplementary appeal to WPB for relief from the named order.

Attention was called by Mr. H. A. Wheeler to the increased advertising rates for the PROCEEDINGS, which were authorized at the previous meeting as the result of a sharp gain in membership.

The recommendation was made to the Board of Directors that H. N. Blackmon, R. K. Honaman, and E. L. Robinson be appointed to the Papers Procurement Committee.

A revised budget for the PROCEEDINGS was recommended to the Board of Directors for approval as a result of the larger number of copies being printed, papers published, and advertisements carried, in addition to the possibility of having to make certain space-saving format changes due to the WPB Paper Limitation Order. A similar budget recommendation was made in the case of the Cumulative Index (1913–1942) in view of the necessity of printing more copies than were originally planned.

Dr. F. E. Terman was recommended to the Board of Directors for appointment as the Institute representative on the Division of Engineering and Research of the National Research Council, Washington.

The conditional appointment of Mr. R. H. Dishington as Institute Representative at the University of Southern California, to succeed Professor J. K. Nunan who is on leave of absence from that university, was also recommended for approval by the Board of Directors.

Dr. Llewellyn reported on the further progress that is being made in revising the list of Institute Representatives on Other Bodies.

Consideration was given to a request from the Subcommittee on Supplementary Finance of the American Standards Association.

Chairman Wheeler reported on an analysis relative to the request of the Chicago Section for enlargement of its official Section territory. On the basis of the report, it was recommended that the Board of Directors approve the addition of certain counties to the present Chicago Section area.

Matters pertaining to the New York and

Buenos Aires Sections were also discussed.

It was stated by Chairman Wheeler that a request had been received from the Associated Radio Technicians of British Columbia concerning the possibility of forming an Institute Section at Vancouver, British Columbia, Canada.

The 1942 Report of the Secretary was discussed and recommended to the Board of Directors for approval.

A discussion of the Rochester Fall Meeting Committee was also held.

Consideration was given to material recently received from National Inventors Council, Washington.

The appointment of F. A. Polkinghorn to the Admissions Committee was recommended to the Board of Directors for approval.

Dr. Llewellyn called attention to the Temporary Facsimile Test Standards report, recently completed by the Facsimile Committee and submitted by J. L. Callahan, chairman of that group. The recommendation was made that the Board of Directors approve the printing of this and other temporary standards during the war period.

Chairman Wheeler stated that the report on the Institute's long-range investment policy was progressing and would soon be available for discussion by the Executive Committee. The appointment of a standing committee on investments was recommended to the Board of Directors.

A request from the American Red Cross was considered.

1943 Rochester Fall Meeting

Sagamore Hotel, Rochester, New York
November 8 and 9, 1943

Program

Monday, November 8

8:30 A.M.

REGISTRATION

9:30 A.M.

TECHNICAL SESSION

"Review of the problem: Demountable Versus Sealed-Off Tubes," by I. E. Mouromtseff, Westinghouse Electric and Manufacturing Company.

"Recent Advances In Klystron Theory," by W. W. Hansen, Sperry Gyroscope Company.

12:30 P.M.

LUNCHEON

2:00 P.M.

TECHNICAL SESSION

"The Design of Intermediate-Frequency Transformers for Frequency-Modulation Receivers," by William H. Parker, Jr., Stromberg-Carlson Telephone Manufacturing Company.

"Vacuum Capacitors," by George H. Floyd, General Electric Company.

4:00 P.M.

COMMITTEE MEETINGS

6:30 P.M.

DINNER

8:15 P.M.

TECHNICAL SESSION

"The Signal Corps Looks to the Engineer," by Lieutenant Colonel Kenneth D. Johnson, United States Army Signal Corps.

Tuesday, November 9

8:30 A.M.

REGISTRATION

9:30 A.M.

TECHNICAL SESSION

"Message of RMA Director of Engineering," by Dr. W. R. G. Baker.

"Operating Characteristics of Ceramic Dielectrics with Constants Over 1000," by R. B. Gray, Erie Resistor Corporation.

"A Chamber of Commerce War Research Committee," by K. C. D. Hickman, Distillation Products, Inc.

12:30 P.M.

LUNCHEON

2:00 P.M.

TECHNICAL SESSION

"Report of RMA Data Bureau," by L. C. F. Horle

"New Low-Loss Ceramic Insulation," by Ralston Russell, Jr., and L. J. Berberich Westinghouse Electric and Manufacturing Company.

"Design of Intermediate-Frequency Systems," by J. E. Maynard, General Electric Company.

4:00 P.M.

COMMITTEE MEETINGS

6:30 P.M.

STAG BANQUET

Toastmaster, R. M. Wise

(Subject and Speaker to be announced later.)

An exhibit of the U. S. Army Signal Corps equipment will be a feature of both days.

ELMER C. GRIMLEY
1891–1943

Elmer C. Grimley of Montreal, president of RCA Victor, Ltd., was drowned on August 3, 1943, while sailing his boat on Lake Memphramagog.

Mr. Grimley was born in Cornwall, New York, on April 15, 1891. From 1908 to 1912 he was an apprentice in the engineering department and a junior draftsman with the Gregg Company, Ltd., at Hackensack, New Jersey. In 1916 he received the B.Sc. degree from New York University. During 1916–1917 Mr. Grimley was a junior accountant with Marwick, Mitchell, Peat and Company in New York and a Lieutenant (J.G.) in the United States Navy from 1917 to 1919. From 1919 to 1935 he was with RCA Victor in the United States and in 1935 he went to Canada to be president of RCA Victor Company, Ltd.

Mr. Grimley was a past president and director of the Radio Manufacturers Association and a director of the Bankers Commercial Corporation. He joined the Institute of Radio Engineers in 1939 as an Associate.

Books

First Principles of Radio Communications, by Alfred Morgan

Published by D. Appleton-Century Co., 35 W. 32 St., New York, N. Y. 353 pages +12-page index+IX pages. 183 figures. 6×8½ inches. Price, $3.00.

The author is a radio engineer who has written a number of popular books on scientific subjects. The purpose of the present work, as outlined in the publisher's notice, (there is no author's preface), is to provide a practical, nonmathematical, introductory course in radio communications for the novice.

The first 141 pages serve to lay a foundation of the broad principles of electrical science as an introduction to the later chapters on strictly radio matters. This preliminary material, which bases the treatment of electrical circuits on the conception of the electron, is presented clearly and in an interesting manner, with a minimum of formulas and only the simplest arithmetic. The diagrams throughout the book are clear and striking and are accompanied by well-chosen captions.

The later chapters on radio apparatus and accessories are also clear, informative, and up to date. The portions dealing with vacuum-tube circuits are less satisfactory, although the subject matter is well chosen and the treatment logical and accurate. The radio amateur and others who already are acquainted with elementary radio circuits may read this portion with profit, but it is not certain that the novice will find it as clear as the rest of the book. It may, however, be doubted whether the simplified method of presentation so successful in the other sections of the next, can be made to yield more than a superficial understanding of the complications of the circuits of a radio receiver, to the reader entirely unfamiliar with the subject, in the space available.

The author is, however, to be congratulated on the production of a concise, comprehensive, and accurate introduction to the chosen subject, which is well suited to serve as an inspiration to the beginner and as an incentive to further study on his part.

FREDERICK W. GROVER
Union College
Schenectady, N. Y.

A Course in Radio Fundamentals, by George Grammer

Published by the American Radio Relay League, West Hartford, Conn. 103 pages. 104 illustrations. $6\frac{1}{2}\times9\frac{1}{2}$ inches. Price, $.50.

"A Course in Radio Fundamentals" suggests that to the adage, "Don't judge a book by its cover" there be added "or its size." In the format of *QST*, of which the author of the book is Technical Editor, its narrow margins and small compact type make its 103 pages easily the equivalent of two or three times that many of the more-conventional variety.

Partaking more of the nature of a syllabus than of a text, it is the objective of this book to organize the approach of the new student of radio to the study of fundamentals as they are presented in "The Radio Amateur's Handbook."* In addition, it describes the apparatus and the procedure for a series of experiments to familiarize the neophyte with the operation of equipment. Examination questions are given at brief intervals to test the effectiveness with which the student is absorbing the knowledge to which he has been exposed. The answers may be found at the back of the book.

Characteristic of the technical view of the editorial group of the American Radio Relay League, the emphasis is placed firmly on matters of practical importance. Theory is used as a basic means to the building and use of equipment and not for mental acrobatics.

The reviewer confesses some bias, having been associated with *QST* for several years, and may thus be excused for hoping that some day *QST* and its companion publications will reserve the term "capacity" for the ability to do work and use "capacitance" to designate the property of a capacitor. "Condenser" can then be returned to the steam plant. This gives an orderly system of nomenclature:

Device	*Property*
Resistor	Resistance
Inductor	Inductance
Capacitor	Capacitance

While it may not be inevitable, it is not uncommon for the amateur to become a professional. Amateur training, therefore, is of importance to the engineering profession as it influences the thinking and terminology of many engineers. Habits learned early in life are hard to change as is indicated by a well-squelched preference developed by the reviewer when an amateur for "picofarad" rather than the cumbersome "micromicrofarad."

This pair of books is undoubtedly the biggest $1.50 worth of technical radio literature offered the newcomer to the high-frequency radio field. It answers completely the question which many engineers face from time to time when advising those who are interested in getting started in radio. It is this aspect which justifies the engineer in buying the books to evaluate them and—not so incidentally—to spend a few hours catching up on some of the branches of the field with which he may not be in daily contact.

HAROLD P. WESTMAN
Secretary, War Committee on Radio
American Standards Association
New York, N. Y.

* PROC. I.R.E., vol. 31, p. 313; June, 1943.

Elements of Radio, by A. Marcus and Wm. Marcus

Published by Prentice-Hall, Inc., 70 Fifth Avenue, New York, N. Y. 684 pages +15-page index+xiii pages. 539 figures. $6\frac{1}{4}\times9\frac{1}{4}$ inches. Price $4.00.

The authors' purpose in writing this book was to provide a basic and elementary course suitable for home study or as a classroom text on the fundamentals of radio for those having no previous experience in physics or mathematics. The method chosen to accomplish this can best be illustrated by a quotation from the preface: "The simple crystal receiver is chosen as embodying all the basic principles of any receiver."

To insure a complete understanding of this simple receiver, the device of the spiral is adopted in presenting the subject matter. Thus, at the first cycle an extremely elementary explanation of the radio is presented—tuning, detection, and reproduction. The next turn around covers the same ground at a slightly higher level. And so on, through the various cycles.

The authors consider the elimination of all formulas and mathematics a "must" and as a consequence, the first half of the book does not contain formulas. In this half of the book the student is taken from the most elementary concepts up through the operation of the superheterodyne receiver. It is interesting to note in the first edition of a new book that many of the diagrams and examples show circuits employing such tubes as the 01A, 27, 24A, and 47 and that the superheterodyne receiver discussed employs an intermediate frequency of 175 kilocycles.

The second half of the book takes up in a more technical manner the consideration of the nature of electricity and the radio wave. Here are introduced for the first time the technical definitions of such terms as inductance, capacitance, resonance, etc. Direct- and alternating-current circuits, electromagnetic waves, radio antennas and transmitters, vacuum-tube amplifiers and oscillators, and other subjects common to a book of this type are covered in this section.

The book incorporates a number of worth-while teaching devices. At the beginning of each chapter problems are set up, the answers to which are developed in the chapter. At the end of each chapter there is a concise summary, or glossary listing the important concepts and definitions covered in the chapter. This is followed by questions and problems.

The twenty-five pages preceding the appendix list demonstrations for each chapter in the book. There are from 1 to 13 suggested beginning with chapter 2. These require equipment which would be readily available in any school with the ordinary equipment in physics with the addition of several old radio sets.

The appendix contains 9 pages of data including eventful dates in radio development, radio symbols and abbreviations, Radio Manufacturers Association color codes, international Morse code, soldering hints, and practical data.

Very little space is devoted to frequency modulation. The discussion of this subject is far too brief and inadequate, being confined to about three pages.

A half page is devoted to facsimile and a 14-page chapter to the cathode-ray tube and its application.

The subject of ultra-high frequency receivers and transmitters is not treated.

Since the book presupposes no earlier training in physics or mathematics on the part of the reader, and contains numerous errors and debatable statements, it will not be of interest to qualified engineers.

W. O. SWINYARD
Hazeltine Electronics Corporation
Chicago, Illinois

Applied Electronics, by Members of the Staff of the Department of Electrical Engineering, Massachusetts Institute of Technology

Published (1943) by John Wiley and Sons, Inc., 601 West 26 St., New York, N. Y. 772 pages, 738 pages+34-page index +xxiii pages. 394 figures, $6\frac{1}{2}\times9\frac{1}{2}$ inches. Price, $6.50.

This third volume in the "Principles of Electrical Engineering Series" by the staff at M.I.T. is a textbook, and its greatest use should be in the classroom. It is, in agreement with its subtitle, a "first course." The practicing engineer will find it a useful reference in the general field. It makes no pretense of being useful to the specialist in his own speciality.

The logical organization of the material will appeal both to the teacher and to the student. The first third of the book is spent inside the electronic tube, studying electron ballistics, emission, conduction through vacuum and gases, and tube characteristics. The last two thirds of the book deals with the circuits closely associated with the tube. It includes chapters on rectifiers, class A amplifiers, class AB, B, and C amplifiers, oscillators, and modulators and detectors.

Each chapter starts with a general description, usually followed by a brief mathematical discussion concerning basic relations or idealized cases. The remainder of the chapter then develops the broader problem. The emphasis marks with which the student usually decorates the important mathematical relations are already printed alongside the equation numbers.

The discussion of some subjects, such as feedback amplifiers, seems very brief, but more complete consideration of special problems can hardly be expected in a first course.

In the discussion of amplifiers, gain and

amplification are used without distinction. It is customary for communication engineers to use amplification for simple ratios and to use gain only when the units are decibels. This practice should be reflected in our textbooks.

In the discussion of units, p. 724, it is stated: "In any electromagnetic system of units, the only unavoidable dimensionless factor which appears because of the geometry is 4π." It is unfortunate that the authors have not noted the existence of a well-known system of units which does avoid 4π and all other dimensionless factors which might enter because of the geometry or for any other reason. The authors are to be complimented on the use of the m.k.s system of units even in its incomplete form, but many will regret that they did not go all the way. A textbook concerned with the development of logical concepts and methods of analysis should distinguish between the effect of geometry in a system of units and its effect in a particular application which may involve those units.

At the end of each chapter is a rather generous list of very appropriate problems. These, together with the excellent general presentation, make this a very "teachable" book.

E. B. Ferrell
Bell Telephone Laboratories, Inc.
New York, N. Y.

Tables of Functions with Formulae and Curves, by Eugene Jahnke and Fritz Emde

Published by Dover Publications, 31 East 27 Street, New York, N. Y. Enlarged, revised edition, 1943. 303+76 pages. 212 figures. $6\frac{1}{4}\times9\frac{1}{2}$ inches. Price, $3.50.

The 1933 edition of this work received immediate recognition as a most valuable handbook of mathematical functions for the use of mathematicians, physicists, and engineers. The 1938 edition was enlarged and improved in some respects, but did not contain one valuable section of the previous edition, entitled "Tables of Elementary Functions." The 1943 edition is a reprint of the entire 1938 edition with the addition of this section, so it contains all the material of both previous editions. It has been published under license of the United States Alien Property Custodian to make this valuable handbook available to the growing army of scientists which has come to be recognized as one of the determining factors in the war. The photo-offset method of reprinting has been employed to minimize the delay in printing and the cost of reproduction, while assuring an exact copy.

The 1938 edition was reviewed in the Proceedings of the I.R.E. of July, 1942, page 353. For the benefit of more recent members, that review is quoted herewith.

"This volume is a monumental collection of short tables, formulas, and curves of a large variety of transcendental functions. Many of the functions are plotted in three dimensions to give a better picture of the functional relations. The contour method of plotting in two or three dimensions is used in some cases. All subjects are well supported by references to more comprehensive specialized publications.

"The text is complete in both English and German, side by side.

"Some of the functions which are of greatest interest to radio engineers may be mentioned. The sine-integral, cosine-integral, and exponential-integral are useful in the analysis of transients in idealized filters. The error integral finds application in thermal agitation.

"Elliptic integrals are represented by one of the best short collections of formulas, curves, and tables; they find application in the inductance of coils, capacitance of rings, and special problems such as linear detection of modulated waves with unsymmetrical sidebands.

"The largest section, about one half of the book, is devoted to an excellent treatment of Bessel functions of integral and fractional orders with real, imaginary, and complex arguments. The essential relations are concisely formulated and illustrated graphically. Special attention is given to asymptotic formulas and to special properties such as the roots of the functions and their derivatives. There are 28 tables of the various cases. This section finds application to countless radio problems, from the high-frequency resistance of wires to the propagation in wave guides and attenuation in piston attenuators. Special applications are found in the transients in filters and the sidebands in frequency modulation."

H. A. Wheeler
Hazeltine Electronics Corporation
Little Neck, Long Island, N. Y.

Principles and Practice of Radio Servicing, by H. J. Hicks

Published by McGraw-Hill Book Company, 330 West 42 Street, New York, N. Y., and London, England. Second edition 1943. 380 pages+11-page index+XII pages. 311 figures and tables. $6\frac{1}{2}\times9$ inches. Price, $3.50.

This book seems to have been prepared as a connecting link between the technical radio theory of the classroom and the practical problems of a radio servicing shop. It may well serve (at some time) to reintroduce the subject of home-radio receiver repairing to the host of technicians who are trained in radio only as to its military aspects.

Starting with brief reviews of electrical and radio fundamentals and the basic principles of tubes and their uses in circuits, the author proceeds with descriptions of test equipment, amplifiers for various uses, power supplies, and detectors. Receiver circuit features and individual equipment components are described, followed by the circuit theory of receivers (including frequency modulation) and a general description of the generally accepted servicing methods. The book has a chapter on public-address systems and suggestions as to the business problems of the servicing game.

Ralph R. Batcher
Hollis, L. I., N. Y.

Introduction to Circuit Analysis, by Abner R. Knight and Gilber H. Fett

Published (1943) by Harper and Brothers, 49 East 33 Street, New York, N. Y. 439 pages+7-page index+XX pages. 219 figures, $6\frac{1}{2}\times9\frac{1}{2}$ inches. Price, $4.00.

This is a very well written book. The explanations are clear and well thought out. There is an excellent use of graphical attack. The comparison of the systems of units is also fortunate. In fact for the purpose for which it is intended it seems like an excellent job.

This reviewer's only quarrel would be with the purpose of the book. It is "intended as a text for the initial and basic course in electrical engineering." As such it presumably follows right after physics and calculus are completed. From this start to cover circuit analysis in "a five-semester hour course" requires a superficiality of treatment that bodes ill for the student's depth of understanding of the subject. Direct-current problems are dismissed in one-half page. This completely neglects the opportunity to give such stimulating problems as the cube which is made up of 1-ohm resistors along each edge and whose resistance between diametrically opposed corners is required. Or some such as the double-track trolley system fed by a single feeder with different voltages at the two ends of the line. It is required to determine the minimum line voltages if the four cars on the line are never allowed to approach each other more closely than one mile on the same track. Such problems as the latter teach the young student to translate physical problems into mathematical language and so become familiar with the calculus as a working tool.

In fact the main criticism of the book is that the authors fall into the common engineering error of regarding calculus as a beautiful woman: lovely to behold but not to be used. For instance, the superposition theorem (p. 288) is not intuitively obvious, but a result of the facts that experimentally certain electrical circuits behave according to a linear second-order differential equation, and that the sums of solutions of such an equation are also solutions of the equation.

Of course what this reviewer decries about the book will appear a virtue to many. And for those to whom this will be true the book should be a very satisfactory one. The choice of material is good. It is a very gratifying to see the teaching of *mesh currents* in elementary courses increasing. Also the use of determinants in the solution of network problems.

The book has surprisingly few errors for a first edition. The most annoying is the misinterpretation of the rationalization of equation 13-12 appearing on page 374, since it distorts the physical facts. The book has evidently been carefully proofread; the format is good.

If I believed in the purpose of the book I should like to have written it.

Knox McIlwain
Hazeltine Electronics Corporation
Little Neck, Long Island, N. Y.

Institute Committees—1943

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Lloyd Espenschied
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L. E. Whittemore
G. W. Willard
William Wilson
C. J. Young
P. D. Zottu

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H. P. Westman
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Harley Iams
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A. F. Pomeroy
J. R. Poppele
Simon Ramo
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P. C. Sandretto
S. A. Schelkunoff
D. B. Sinclair
Karl Spangenberg
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Dayton Ulrey
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Irving Wolff
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B. E. Shackelford
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H. F. Dart
Beverly Dudley
W. J. Ponsolle

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May 1, 1943, to May 1, 1944

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R. F. Guy
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Pierre Mertz
G. D. Robinson
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M. G. Crosby
W. L. Everitt
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H. B. Marvin
D. E. Noble
B. E. Schackelford
D. B. Smith
L. P. Wheeler
W. D. White

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W. F. Cotter
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H. B. Fischer
H. C. Forbes
D. E. Foster
David Grimes
J. K. Johnson
Garrard Mountjoy
H. O. Peterson
E. K. Stodola
A. E. Thiessen
H. P. Westman
R. M. Wilmotte

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C. R. Burrows, *Chairman*

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J. L. Barnes
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W. A. Fitch
G. D. Gillett
H. R. Mimno
K. A. Norton
H. O. Peterson
H. P. Thomas
W. D. White

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I. J. Kaar
G. G. Muller
H. M. Turner

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H. M. Turner, *Chairman*

R. R. Batcher
M. R. Briggs
R. S. Burnap
C. R. Burrows
J. H. Dellinger
E. T. Dickey
H. S. Knowles
O. T. Laube
Allen Pomeroy
E. W. Schafer

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P. C. Goldmark
T. T. Goldsmith, Jr.
A. G. Jensen
A. F. Murray
R. E. Shelby
D. B. Sinclair
D. B. Smith

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Raymond Asserson
M. R. Briggs
W. W. Brown
Harry Diamond
W. S. Duttera
Sidney Frankel
F. A. Gunther
W. E. Jackson
R. W. P. King
J. F. Morrison
J. C. Schelleng
Robert Serrell
D. B. Sinclair
W. D. White
J. E. Young

Institute Representatives in Colleges—1943

Alabama Polytechnic Institute: Woodrow Darling
Alberta, University of: J. W. Porteous
Arkansas, University of: P. K. Hudson

British Columbia, University of: H. J. MacLeod
Brooklyn, Polytechnic Institute of: F. E. Canavaciol

California Institute of Technology: S. S. Mackeown
California, University of: L. J. Black
Carleton College: C. A. Culver
Carnegie Institute of Technology: R. T. Gabler
Case School of Applied Science: P. L. Hoover
Cincinnati, University of: W. C. Osterbrock
Colorado, University of: J. M. Cage
Columbia University: J. B. Russell
Connecticut, University of: P. H. Nelson
Cooper Union: J. B. Sherman
Cornell University: True McLean

Drexel Institute of Technology: R. T. Zern
Duke University: W. J. Seeley

Florida, University of: P. H. Craig

Georgia School of Technology: M. A. Honnell

Harvard University: E. L. Chaffee

Idaho, University of: Hubert Hattrup
Illinois Institute of Technology: P. G. Andres
Illinois, University of: H. J. Reich
Iowa, University of: R. C. Kent

Johns Hopkins University: Ferdinand Hamburger, Jr.

Kansas State College: Karl Martin
Kansas, University of: G. A. Richardson

Lawrence Institute of Technology: H. L. Byerlay
Lehigh University: C. B. Brenecke
Louisiana State University: Taintor Parkinson

Maine, University of: W. J. Creamer, Jr.
Manhattan College: J. F. Reintjes
Maryland, University of: G. L. Davies
Massachusetts Institute of Technology: W. H. Radford and E. Guillemin
McGill University: F. S. Howes
Michigan, University of: L. N. Holland
Minnesota, University of: O. A. Becklund
Montana State College: G. J. Fiedler

Nebraska, University of: F. W. Norris
Newark College of Engineering: Solomon Fishman
New York University: Philip Greenstein
North Dakota, University of: E. J. O'Brien
Northeastern University: G. E. Pihl
Northwestern University: A. B. Bronwell
Notre Dame, University of: H. E. Ellithorn

Ohio State University: E. C. Jordan
Oklahoma Agricultural and Mechanical College: H. T. Fristoe

Pennsylvania State College: G. H. Crossley
Pennsylvania, University of: C. C. Chambers
Pittsburgh, University of: R. C. Gorham
Princeton University: J. G. Barry
Purdue University: R. P. Siskind

Queen's University: H. H. Steward

Rensselaer Polytechnic Institute: H. D. Harris
Rice Institute: C. R. Wischmeyer
Rose Polytechnic Institute: G. R. Schull
Rutgers University: J. L. Potter

Southern Methodist University: R. E. Beam
Stanford University: Karl Spangenberg
Stevens Institute of Technology: F. C. Stockwell

Texas, University of: E. W. Hamlin
Toronto, University of: R. G. Anthes
Tufts College: A. H. Howell

Union College: F. W. Grover
United States Naval Academy: G. R. Giet
Utah, University of: O. C. Haycock

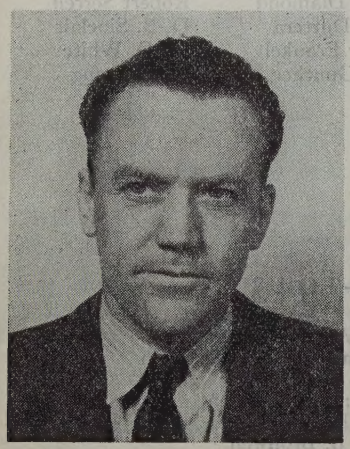
Virginia, University of: L. R. Quarles
Virginia Polytechnic Institute: R. E. Bailey

Washington, University of: A. V. Eastman
Washington University: Stanley Van Wambeck
Wayne University: G. W. Carter
Western Ontario, University of: G. A. Woonton
West Virginia University: R. C. Colwell
Wisconsin, University of: Glenn Koehler
Worcester Polytechnic Institute: H. H. Newell

Yale University: H. M. Turner

Contributors

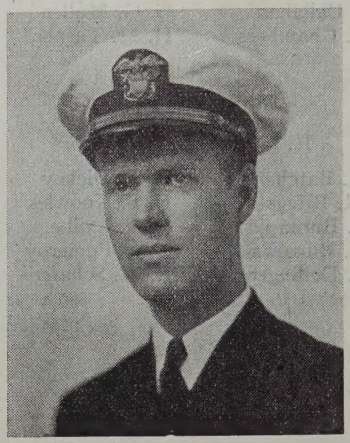
R. A. Bierwirth was born at Anthon, Iowa, in 1901. He received the B.S. degree in electrical engineering from Iowa State College in 1925, and the M.S. degree in electrical engineering from Union College in 1928. Mr. Bierwirth was employed in the radio engineering department of the General Electric Company at Schenectady from 1925 to 1930; in the engineering department of RCA Manufacturing Company at Camden from 1930 to 1941; and is now engaged in research work on industrial applications of radio frequency at the RCA Laboratories at Princeton.



R. A. Bierwirth

❖

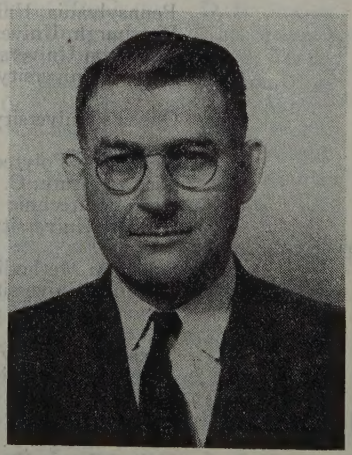
Charles W. Harrison, Jr. (A '36) was born in Virginia in 1913. The years 1932–1936 were spent in the U. S. Naval Academy Preparatory School and in the Academy. In 1939 Mr. Harrison received the S.B. degree in engineering and in 1940 the degree of electrical engineer from the University of Virginia. In 1942 he was graduated with the S.M. degree in communication engineering from the Cruft Laboratory, Harvard University, and during the summer was a student at the Massachusetts Institute of Technology. His experience includes amateur, naval, and broadcast station operation, as well as research work at the Navy Department in Washington. He is at the present time a member of the teaching staff of the Cruft Laboratory.



C. W. Harrison, Jr.

❖

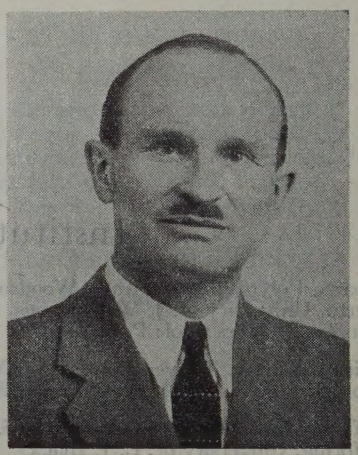
Cyril N. Hoyler (A '35) was born on August 8, 1905, at Edmonton, Alberta, Canada. He received the B.S. degree from Moravian College in 1928 and the M.S. degree in physics from Lehigh University in 1935. After teaching mathematics and German for one year in the public schools of Irvington, New Jersey, he was invited to join the faculty of Moravian College in 1929 to develop a department of physics at that institution. Mr. Hoyler held that position until 1941 when he joined the research laboratories of the RCA Manufacturing Company, Inc., and is now located in the RCA Laboratories at Princeton, New Jersey. He has held an amateur operator's license since 1930 and a broadcast operator's license since 1940.



Cyril N. Hoyler

❖

Ronold King (A '30) was born on September 19, 1905, at Williamstown, Massachusetts. He received the B.A. degree in 1927 and the M.S. degree in 1929 from the University of Rochester and the Ph.D. degree from the University of Wisconsin in 1932. He was an American-German exchange student at Munich from 1928 to 1929; a White Fellow in physics at Cornell University from 1929 to 1930; and a Fellow in electrical engineering at the University of Wisconsin from 1930 to 1932. He continued at Wisconsin as a research assistant from 1932 to 1934. From 1934 to 1936 he was an instructor in physics at Lafayette College, serving as an assistant professor in 1937. During 1937 and 1938 Dr. King was a Guggenheim Fellow at Berlin. In 1938 he became instructor in physics and communication engineering at Harvard University, advancing to assistant professor in 1939 and to associate professor in 1942.



Ronold King

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For biographical sketches of George H. Brown, E. W. Herold, and L. Malter, see The Proceedings for August, 1943.